

Experiments with 3D Printed Coil Forms

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February 22, 2024

The engineer who has occasion to calculate an inductance is likely to be overwhelmed by the very wealth of formulas offered him.

Frederick W. Grover, 1946

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1 Why?

Precision inductors are a bit of an anachronism - for small values, parasitic inductance may predominate. The inductance calculation may suffer from the geometry necessary for mechanical stability and so on. Tuned circuits relying on fixed inductors usually have variable capacitors to make up for this lack. This may not be an option hence the need for inductors with predictable and measurable values.

Looking for precision inductors to complement the “precision” capacitors of a previous study, I went on-line to find the number of turns and geometry for my needs - $1.6 \mu h$ in a reasonable size with a modest Q . Using one of the equations I computed the number of turns, the radius and length. I printed the form using PLA, a reasonable dielectric, and got no where near the desired result. Many repeated attempts with different PLA densities, diameters, turns and lengths I finally got within 6% of the desired result but with much waste and frustration. It was clear that the geometry of the printed form was not that expected by the equation or that the equations had unspecified restrictions. The passage of years has done little to ameliorate Grover’s 75 year old observation.



Figure 1: Some of those that were not close enough.

3D printing opens up possibilities for inductors beyond that envisioned by the approximation in the ARRL handbook[2]¹. Here \mathcal{L} is the inductance in microHenries, d the coil diameter, n the number of turns and l the coil length in inches.

$$\mathcal{L} = \frac{d^2 n^2}{18d + 40l} \quad (1)$$

¹In the text that follows, we abbreviate this equation **ARRL**

Rather than try and capture the physics behind this in an all encompassing equation, I decided on a number of experiments to capture the variables along a single dimension of change and fit multi-variable polynomials to the experimental results. This is accomplished by a multi-process, genetic optimization program. It tries billions of solutions on 150+ measured coils and finds a best fit within the ranges of values allowed.

A second program converts your inductance requirements into an STL file for 3D printing. You give it the inductance you want, set ranges for the size, wire gauge, and turns, then, after a while, an STL file is created.

2 Existing Solutions

The inductance measure is named after Joseph Henry, the units we use are micro Henries or μH , one millionth of A henry. You can see him in front of the Smithsonian and as the last named mountain range in the US (in southern Utah).

Accurate solutions are feasible using Maxwell's equations and complex geometric descriptions. For example, see *Inductance Calculations* [6] or *Inductance Loop and Partial* [13]. We propose to accept simple polynomial approximations derived by experiment as have the approximations described below.

The number of different equations on the net was truly surprising until an offhand comment about them being derived from experiments rather than physics was an eye opener. It's clear that each approximation has unadvertised limits on the coil parameters. The basic parameters for a cylindrical single layer coil include:

- n the number of turns (somewhat lacking in detail),
- l the coil length (somewhat lacking in detail),
- r the coil radius to center of the wire,
- A the area of the core (somewhat lacking in detail),
- μ_r the relative permeability of the core.

After my experiments, I've identified a few other parameters that influence inductance.

- **Parasitic Geometry** The coil may be more complex than a simple helix.
- **Wire Gauge** Even with the same number of turns on the same length, the wire gauge affects the inductance.
- **Humidity** Most 3D printed plastics are hygroscopic. The inductance does vary somewhat with humidity.

- **LCR meters** No two of these read the same values.

Figure 2 shows the kind of coil we're contemplating and some of its troublesome characteristics. Using any standard air coil approximation for this geometry is thwarted by the following:

1. The coil form extends to $\frac{1}{2}$ the wire radius - not air.
2. There's an extra small turn at each end that affects the inductance.
3. The coil core will not be a uniform cylinder. It may have a hole and the plastic interior may not be solid. The relative permeability of plain PLA is not known though probably very close to 1. 3D printing of plastic with relative permeability greater than 1.0 is affected by the density and geometry of the fill.

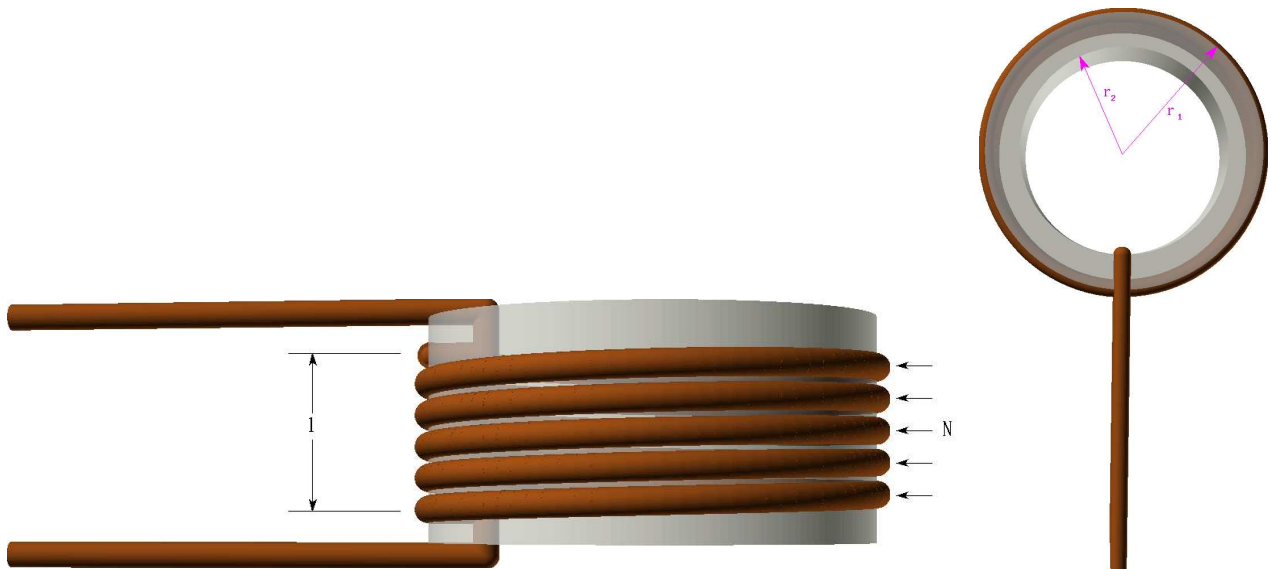


Figure 2: Important coil dimensions.

Here l is the distance between the center of the end of the final turns and the coil area A :

$$A = \pi(r_1^2 - r_2^2) \quad (2)$$

A is only important if μ_r is not 1.0.

3 Sources of Error

Inductance measurements are inexact at best. There are 3 important ones [7].

1. Inductance,
2. Q,
3. Self Resonant Frequency (SRF)

I'll report on all 3 though some of the measurements are *ad hoc*. A few sources of error were identified after the measurements.

Test Leads The bench top East Tester has Kelvin style probes to minimize parasitic external capacitance and inductance. The East Tester clips are gold plated to make good contact with the inductor wires. However, the contacts are scalloped and flat on the top and the spring load small. The Keysight alligator clips are sharp and the spring load large, the leads short but not Kelvin but seem to give more consistent results.

Measurement Device An East Tester 4110 and a Keysight U1733C were used to measure inductance and Q - the results were consistently different. A nano VNA measured SRF.

Inductor wire resistance We're dealing with very low DC resistances, almost always less than one ohm so any dirt, left over enamel or finger grease alters the measurement of Q. We attempt to minimize these errors with frequent cleaning and repeated measurements.

Inductor lead length Where possible, the inductor lead lengths have been trimmed to 1" from the inductor surface and the insulating enamel removed by file and sand paper. This is a particular problem for inductors with few turns. To minimize the error a test stand forces the Kelvin probes and inductor to a fixed position.

Environment Measurements were made over the course of 16 months. During winter, the temperature and relative humidity were fairly constant. 65 degrees Fahrenheit and 28-33% relative humidity. However, for a few weeks, a series of wet winter storms raised the relative humidity to near 40% which may have affected the measurements. A further set of experiments subjected some coils to 100% humidity for at least 2 days and their inductance measured with minimal change.

Calibration I made 30 measurements of each inductor, different groups spread over a few days. The meter was calibrated once for each group and measurements made in different order to account for any changes from previous groups. In no case were more

than 15 measurements made before re-calibration. Still, the inductance varied over the course of a day, usually increasing until the early afternoon and then decreasing later in the day. The meter was allowed to “warm up” for 30 minutes before any measurements were taken.

Nearby Stuff During calibration the large body of mostly water was moved at least 10’ away from the probes. Test coils were at least 12 inches from the test fixture. The test surface was composite granite. There are numerous sources of error - unknown metal nearby, the 110v power cord for the meter, LED lighting, arrangement of the probe wires, and so on.

Physical Dimensions On low cost 3D printers with default settings, resolution is a problem. The wire channels are not perfectly round and are composed of a number of cylinders subtracted from the coil cylinder. This is a particular problem for large diameters and small wires. Likewise, printer age may lead to sloppy edges and cylinder diameter variance.

Filament Shrinkage The common belief is that PLA shrinks 2% in the X and Y axes and nothing in the Z. This was factored into some of the early experiments but further experiments with a new printer and slicer challenge this assumption.

3.1 What is a turn?

A turn has varying definitions. In [1] a straight wire is defined as $N = 1$. Others seem to define a single turn as shown in Figure 3 but this really appears to have the inductance of somewhat more than half a turn. Multiple turns have the same issue at the ends as shown in Figure 4 though the results have less influence with more turns.

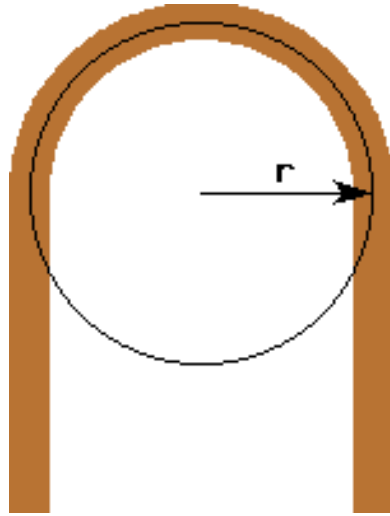


Figure 3: Single turn.

3.2 Some Approximations

The ARRL handbook approximation in equation 1 is reasonable as long as the coil aspect ratio is reasonable, the turns are close together and your core has μ_0 permeability. This, and other equations work better with the geometry in Figure 4.



Figure 4: Air core coil geometry.

Notice the difference between Figure 2 and Figure 4. The air coil is missing $\frac{1}{2}$ a turn but doesn't have the curl to hold the wires in place. This leads to some of the differences that we discover later on.

Another approximation caters to low and high aspect ratio air coils [14]. In what follows, we will call this equation **Russian**. For dimensions in millimeters:

$$\mathcal{L} = \begin{cases} l > r & \frac{\left(\frac{d}{10}\right)^2 n^2}{4.5d+10l} \\ l \leq r & \frac{\left(\frac{d}{10}\right)^2 n^2}{4d+11l} \end{cases} \quad (3)$$

The most common equation appears to come from “Basic AC Circuits” [5]. The Encyclopedia Magnetica provides a formula for non-air core inductors [10, 1]. Here $\mu_0 = 4\pi 10^{-7}$ and μ_r is the relative permeability of the core material. We will use equation 4 to compute the permeability of various inductor cores. In what follows we call this equation **Basic AC**.

$$\mathcal{L} = \frac{n^2 \mu_0 \mu_r A}{l} \quad (4)$$

The following equation from [3] supposedly is accurate to 1% when $l > 0.67r$. We call equation 5 **RFC** and the units are millimeters and the result in microHenries.

$$\mathcal{L} = \frac{0.0394r^2 n^2}{9r + 10l} \quad (5)$$

While reference [8] for $l > 0.8r$ has the same approximation expressed as a function of μ_0 . For most results it corresponds closely to equation 5. We call this equation **RF1**.

$$\mathcal{L} = \frac{\pi r^2 \mu_0 n^2}{9r + 10l} \quad (6)$$

Another on-line tool from EEWeb includes the wire diameter w [4] but unfortunately, does not allow variable wire spacing. The on-line solver appears to have a bug. In metric units we have:

$$\mathcal{L} = n^2 \mu_0 \mu_r r \left(\ln\left(\frac{8d}{w}\right) - 2 \right) \quad (7)$$

In the code this is called **EEWeb**.

Claimed more accurate is Lundin’s approximation of Nagaoka’s tables [9] 90 and avoiding the elliptic integrals of Miller’s solution.

$$\mathcal{L} = \begin{cases} 2r \leq l & \frac{\mu_0 n^2 \pi r^2}{l} \left\{ f_1\left(\frac{4r^2}{l^2}\right) - \frac{4}{3\pi} \frac{2r}{l} \right\} \\ 2r > l & \mu_0 n^2 r \left\{ \left[\ln\left(\frac{8r}{l}\right) - \frac{1}{2} \right] f_1\left(\frac{l^2}{4r^2}\right) + f_2\left(\frac{l^2}{4r^2}\right) \right\} \end{cases} \quad (8)$$

where:

$$f_1(x) = \frac{1 + 0.383901x + 0.017108x^2}{1 + 0.258952x}, 0 \leq x \leq 1 \quad (9)$$

$$f_2(x) = 0.093842x + 0.002029x^2 - 0.000801x^3, 0 \leq x \leq 1 \quad (10)$$

notice the restriction on x . In the code, this is called **LUNDIN**.

The final word appears to be Miller's solving elliptic integrals of the first and second kind [12]. However, it does not account for wire size. For radius r , axial length l and n turns, we have:

$$d = \sqrt{4r^2 + l^2} \quad (11)$$

$$k = \frac{2r}{d} \quad (12)$$

$$K = \frac{1}{3\pi} \left(\frac{dl}{r^2} (F(k) - E(k)) \right) + \frac{4d}{l} E(k) - \frac{8r}{l} \quad (13)$$

The inductance in Henries is:

$$\mathcal{L} = \frac{\mu_0 n^2 \pi r^2}{l} K \quad (14)$$

The elliptical integrals are solved by iteration on k .

$$\begin{aligned} a_0 &= 1 \\ b_0 &= \sqrt{1 - k^2} \\ c_0 &= k \end{aligned} \quad (15)$$

with the recurrence relations:

$$\begin{aligned} a_n &= \frac{a_{n-1} + b_{n-1}}{2} \\ b_n &= \sqrt{a_{n-1} b_{n-1}} \\ c_n &= \frac{a_{n-1} - b_{n-1}}{2} \end{aligned} \quad (16)$$

and run this until $c_n = 0$ or the number of bits in the values of the variables.

If the number of iterations was m , then we have:

$$F(k) = \frac{\pi}{2a_m} \quad (17)$$

$$E(k) = F(k) \left(1 - \frac{1}{2} \sum_{i=0}^m 2^i c_i^2 \right) \quad (18)$$

Normally the series converges quickly. In what follows, this will be called the **MILLER** solution.

I compare these values against the actual data and get the RMS error for each. For a set of m measured values L_i and the computed value M_i the error is:

$$rms = \sqrt{\frac{\sum_i^m \left(\frac{L_i - M_i}{M_i}\right)^2}{m}} \quad (19)$$

Polynomial regression fits are not good predictors outside the data set they fit. The higher the degree, the worse this gets. As a consequence every one of the new equations will have an expressed set of limitations.

4 What's Important?

I decided early on that the number of turns and the ability to mount the coil on a PC card were most important.

Air core inductors are relatively simple to model² but difficult to constrain in practice. A computer program can swiftly search a great range of parameters to get close to the target inductance L . But is this the most important? What about Q? What about size? How much current? This is a typical optimization problem. We assign a $0 \rightarrow 1$ importance to some normalized importance measures $0 \leq M_i \leq 1$ and their assigned importance $0 \leq I_i \leq 1$. The optimal solution is the largest value of:

$$F = \sum_i M_i I_i \quad (20)$$

I've identified a few of these, they read somewhat like a political questionnaire.

Inductance How important is the exact inductance L_t ? 0 - don't care (so why bother), 0.5 - somewhat important, 1 very important. The computed inductance can range quite largely so we arbitrarily set the range at ± 0.2 :

$$\begin{aligned} L_{low} &\leq L_t \leq L_{high} \\ \text{where :} \\ L_{low} &= L_t - \frac{L_t}{5} \\ L_{high} &= L_t + \frac{L_t}{5} \end{aligned} \quad (21)$$

If the inductance importance is I_t , the computed inductance L , then our value is:

²This is a white lie.

$$F_i = \begin{cases} 0 & L_t \leq L_{low} \\ I_l \frac{|L_t - L|}{L_{high} - L_{low}} & L_{low} < L_t < L_{high} \\ 0 & L_t \geq L_{high} \end{cases} \quad (22)$$

Aspect ratio How important is it that the ratio $\frac{l}{r}$ be close to some value a ? Here we have 3 values required $a_{low} \leq a \leq a_{high}$ that can be asymmetrical. For a value a_i :

$$F_i = I_a \begin{pmatrix} a_i \leq a & \frac{a - a_i}{a - a_{low}} \\ a_i > a & \frac{a_i - a}{a_{high} - a} \end{pmatrix} \quad (23)$$

Q How important is Q? 0 - very important to be low Q, 1 - very important to be high Q. A range is given $Q_{low} < Q_{high}$ so for a Q in this range with importance Q_i we have equation 24.

$$F_i = 2(Q_i - .5) \left(\frac{2Q}{Q_{high} - Q_{low}} - 1 \right) \quad (24)$$

Turns I've limited to single layer coils. Allowing many turns decreases the inductor's Q by increasing the DC resistance, decreasing limits the space. If the turns are limited $T_{low} < T_{high}$ and the importance is T_i with 0 being very important to be few turns, 1 important to have more turns, we have equation 25.

$$F_i = 2(T_i - 0.5) \left(\frac{2T}{T_{high} - T_{low}} - 1 \right) \quad (25)$$

5 Some Experiments

Clearly we can't explore the complete inductor parameter space, instead, we try a few parameters singly. We record the inductance and sometimes Q of an actual coil and attempt to fit a linear equation or low degree polynomial to the result. These include the obvious:

1. The number of turns.
2. The coil diameter.
3. The coil core composition.
4. The coil core fill percentage.
5. The wire gauge.

6. The measurement device.

7. Humidity.

From these we attempt to derive general equations and then verify with actual construction. The data is stored in comma separated variable files with a header line and comments prefixed with double slashes (*//*). Table 22 on page 75 lists the files and their contents.

Finally, we attempt to derive a multivariate polynomial that best fits the range of coils using the variables of:

Radius Coil radius in meters.

Length Coil length in meters.

Wire Size Wire radius in meters.

Turns Number of turns.

I do not attempt to include μ_r in the general polynomial solutions.

5.1 Experimental Apparatus

This consists of a MakerGear M2 3D printer (later a LULZBOT TAZ Workhorse), an East Tester ET4410 LCR meter, temperature and humidity sensor, and test stand. A second LCR, the Keysight U1733C was used on the spacing experiment for comparison.

It was found the the coil position relative to the test connectors needed to be uniform across all the tests. A 3D printed stand in Figure 5 holds both clips in a fixed position. Three different stands for different coil lengths were printed. The Keysight's leads are too short to use these stands.

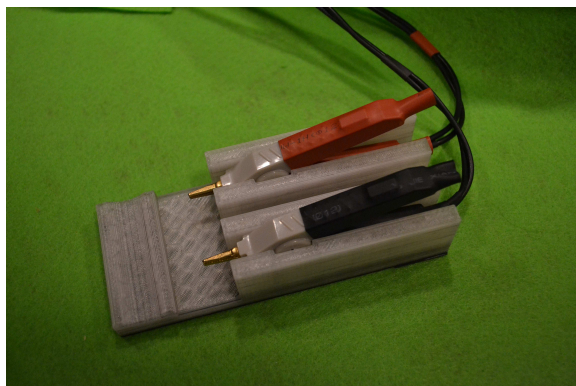


Figure 5: Holder for testing coils.

Both meters are calibrated before every group of tests. The output frequency is 100 kHz at 1 volt. The East Tester instrument does appear to drift during the day even after calibration. Hence 30 measurements spread over three months were taken for all tests. As noted in [7] inductance measurement can vary widely across instruments and test frequencies.

The MakerGear M2 3D printer is capable of printing most of the low temperature PLA filaments. However, ABS, requires a higher temperature than it can maintained and delamination is a problem. The vertical layer is about .3mm or about 0.012" which limits the spiral indent on a form to about 26 gauge wire though some experiments used as small as 28 gauge. The newer printer has a vertical resolution of .15mm if you're willing to wait and some of the final coils were printed at this resolution (anything in lime green).

I printed 4 types:

Plain PLA Polylactic Acid, various colors which all seem to behave the same. A test of the dielectric values indicated they're all the same.

Conductive PLA It appears to have nearly the same permeability as plain PLA but has a much different dielectric value. This is Composite Conductive PLA CDP11705 from Protoplant.

Composite Iron PLA PLA infused with powdered iron of some sort. It does have a much different permeability value than plain PLA. This came from Protoplant, FE11705.

PVA Polyvinyl Alcohol support material - used to wind coils but then dissolved in water for an air core. This is not the panacea that one might hope.

5.2 The Number of Turns

The number of turns is the most important consideration. I printed 10 coil forms - 1" in diameter, with a .125" cylinder wall in plain PLA. The cylinders were filled at 50% though this matters little as the walls are quite thin. Coils from 1 to 30 turns were created as shown in Figure 6. The coils are designed to rest on the test stand at a fixed position.

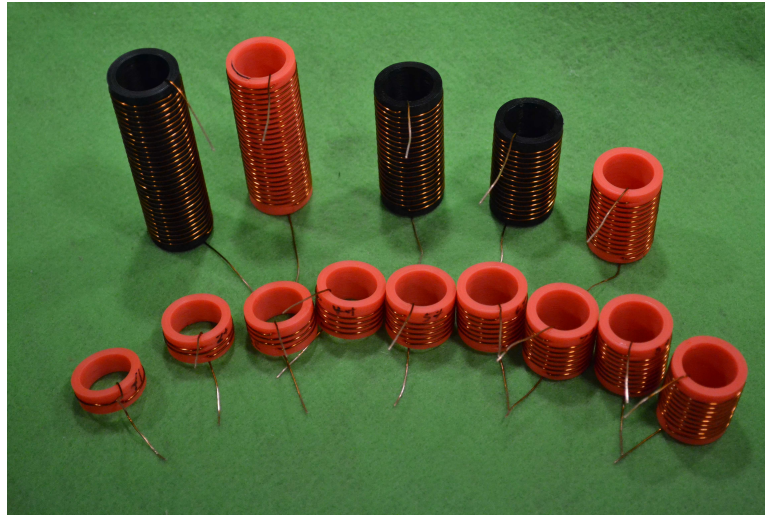


Figure 6: Turns testing coils. 1-10 turns front, 30-12 back.

The values range from about 20 nanoHenries to $6.75 \mu H$ though the smaller values are near the resolution of the meters. I measured inductance of each coil 30 times and recorded these as shown in Figure 7, *turns/turns.csv* and *turns/highaspect.csv* on page 75.

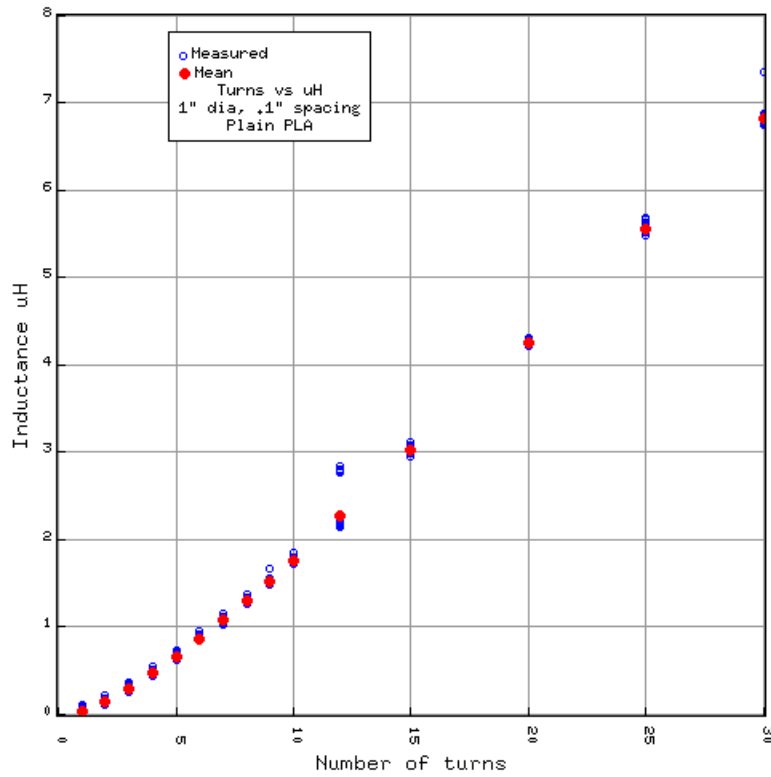


Figure 7: 1 to 30 turns inductance, 1" diameter, .1" spacing #22 wire

A linear equation fits these really well.

$$L = -0.675068 + 0.248447x \tag{26}$$

Equation	RMS error
ARRL	0.23 μ H
Russian	0.19 μ H
RFC1	0.24 μ H
RF1	0.23 μ H
Lundin	0.22 μ H
Linear	0.0009 μ H

Table 1: Linear fit on 1" diameter spacing

Remember that this applies only to PLA coils 1" in diameter with 0.125" wall thickness,

0.1" spacing and #22 wire. We're going to make the assumption that the curve extends beyond three inches though this is dangerous.

6 Permeability of Core Material

The Protoplant iron composite PLA material is interesting because it can increase the Q of a coil without increasing the wire size. However, we don't know what the relative permeability of this material is. Furthermore, it can be printed with varying percentage of fill. I printed five 10 turn coils, 1" in diameter with a 0.175" hole. The fill percentages were 10, 30, 50, 70, and 100:

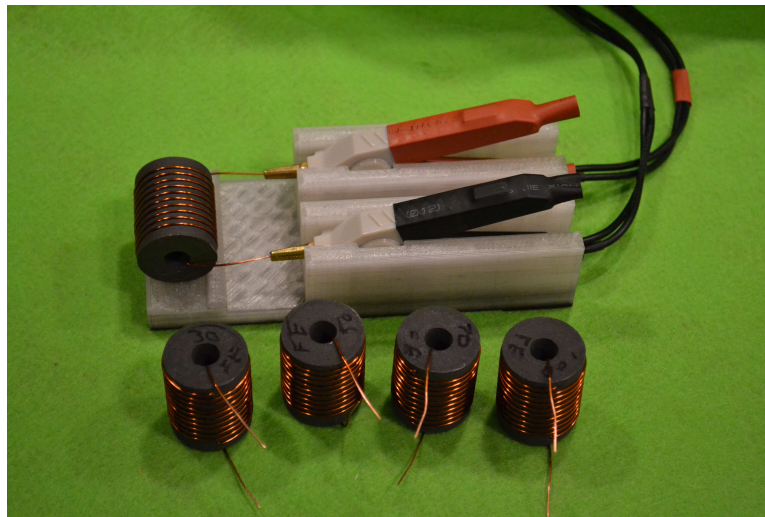


Figure 8: Iron composite PLA coils with varying fill.

The inductance increases linearly with increasing density and all other values being constant. See data file *fepla/fepla175.csv* on page 75.

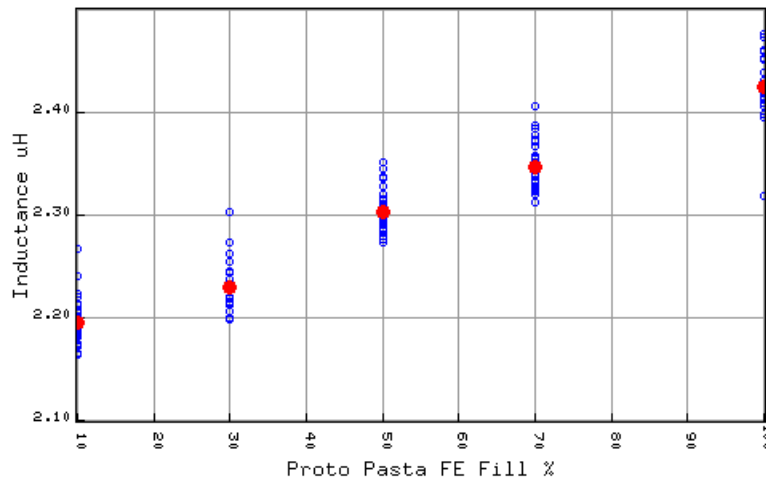


Figure 9: Inductance iron composite PLA fill percentage

Using equation 4 and optimizing for the best μ_r gives us Figure 10.

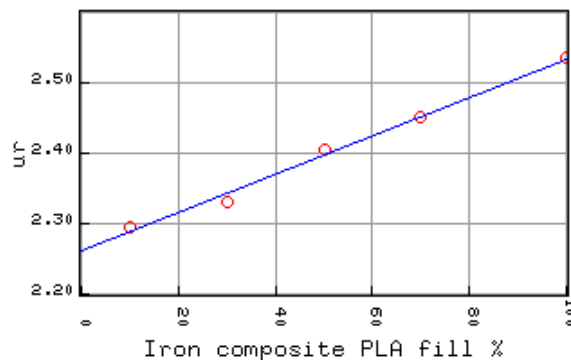


Figure 10: Inductance iron composite PLA fill percentage vs μ_r

The linear equation provides an approximation for the percent fill p for this A.

$$\mu_r = 0.00273p + 2.261 \quad (27)$$

Repeating this experiment with plain and conductive PLA indicates that the fill percentage for these plastics does not affect permeability.

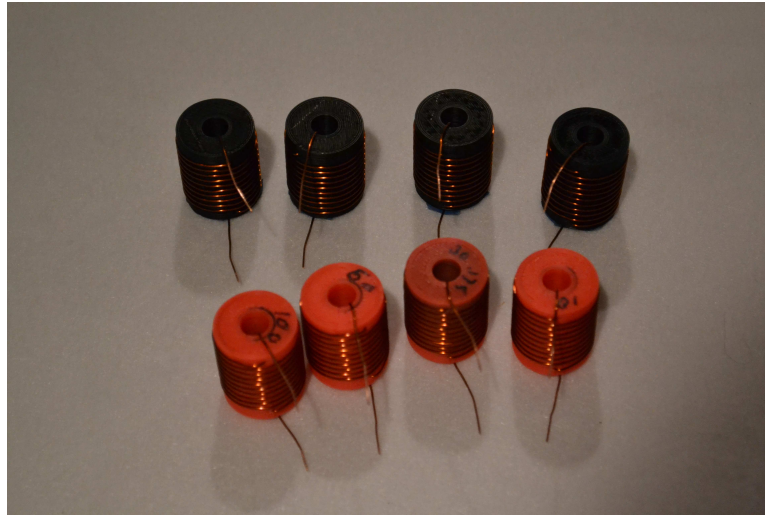


Figure 11: PLA and conductive PLA test coils.

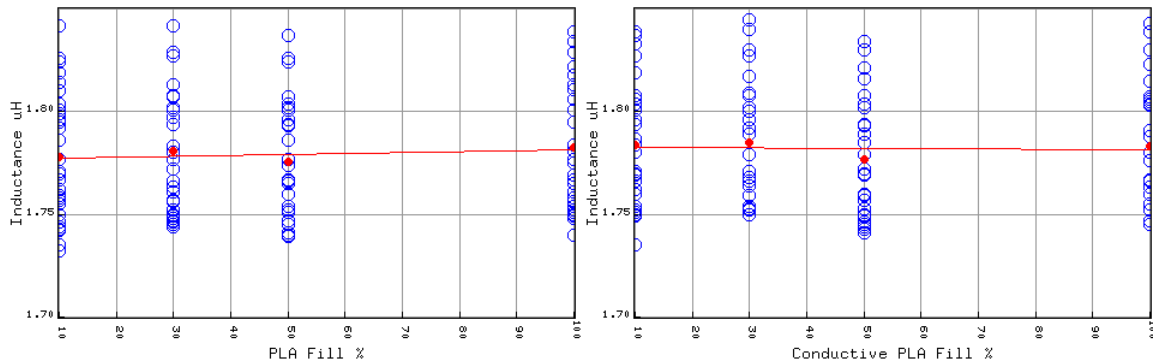


Figure 12: Fill percent vs inductance, PLA, conductive PLA

Working through equation 4, μ_r appears to be between 1.65 to 1.875 for both plastic types. The linear-least-squares fit on the mean in Figure 12 is constant within the limitations of the LCR meter³ Is this correct? My guess is that it's too high because the coil has the extra loops on the end.

Next I repeated the number of turns experiment in Figures 6-7 but with 20% fill iron composite PLA.

³The apparent wide spread is a consequence of the narrow value range and small inductance.



Figure 13: Iron composite PLA coils, 1 to 30 turns

These coils have the same dimensions and turns of those in Figure 6 but a 20% fill percentage - probably meaningless as the walls are mostly solid. The composite does have an effect on inductance as shown in Figure 14.

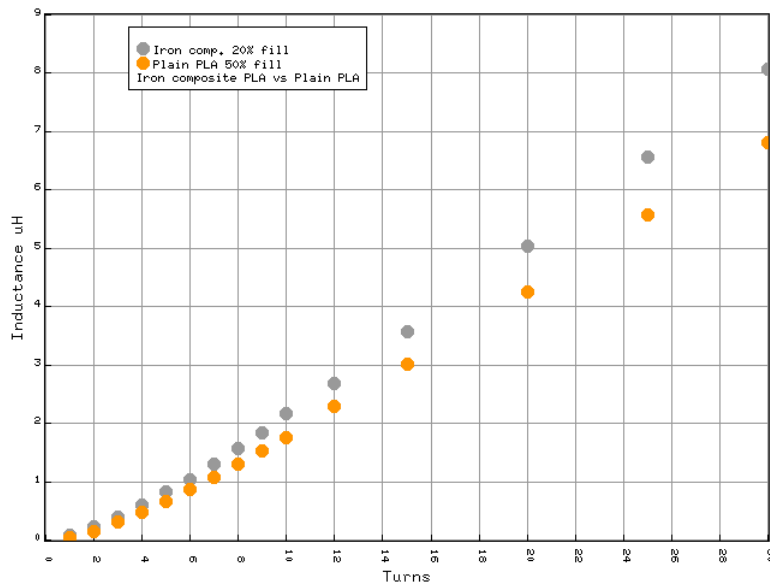


Figure 14: Inductance comparison plain PLA vs Iron composite PLA

Except for small coils, the inductance multiplier for 0.125" wall thickness, 0.1" turn spacing, #22 wire, is about 1.2. That wasn't too exciting. I replaced the .125" wall with a .25" wall with 100% fill and printed the first few coils, unfortunately running out of the expensive filament before completion. Here the multiplier is about 1.4 for bigger coils, still not very exciting but something to contemplate.⁴

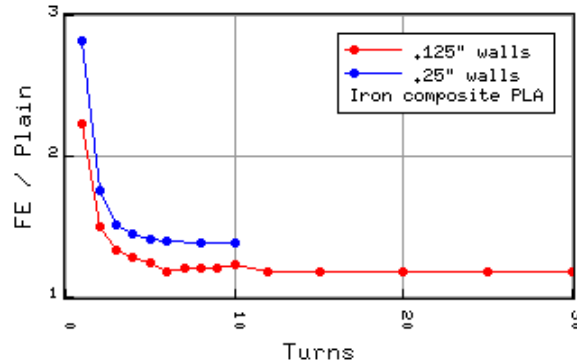


Figure 15: Multiplier for 0.125" and 0.25" wall Iron Composite PLA

7 Turn Spacing

The spacing between turns also matters. 3D printing allows us to experiment with turn spacing. In general, for the same number of turns, the closer the spacing, the higher the inductance. In Figure 16 are some of the coils used in this experiment - 10 turns with a center to center wire spacing of 0.03" to 0.1".

⁴Data still being taken, this from about 15 samples.

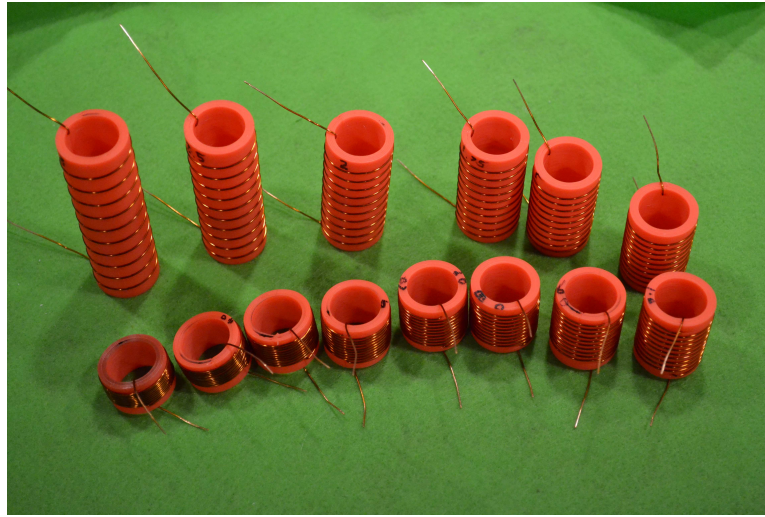


Figure 16: The forms from .3" to 3", 10 turns 1" diameter, #22 wire.

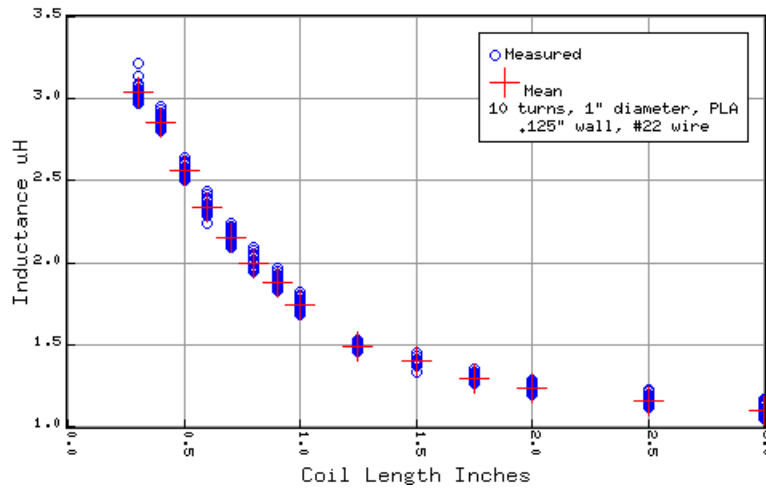


Figure 17: Length vs inductance - 10 turns.

Let's see how the different equations 1, 3, 4, 5, and 6 stack up against this experiment. The best μ_r is computed by trying values from 0.5 to 5 with the best RMS error at $\tilde{1}.422$. We also try different polynomial curve fits up to 10 degrees - the best is 5 with the results shown in Figure 18. The results are much better than any of the approximations **for this range of lengths**.

$$L = 0.0432822l^5 - 0.299272l^4 + 0.480952l^3 + 0.876574l^2 - 3.34232l + 3.97903 \quad (28)$$

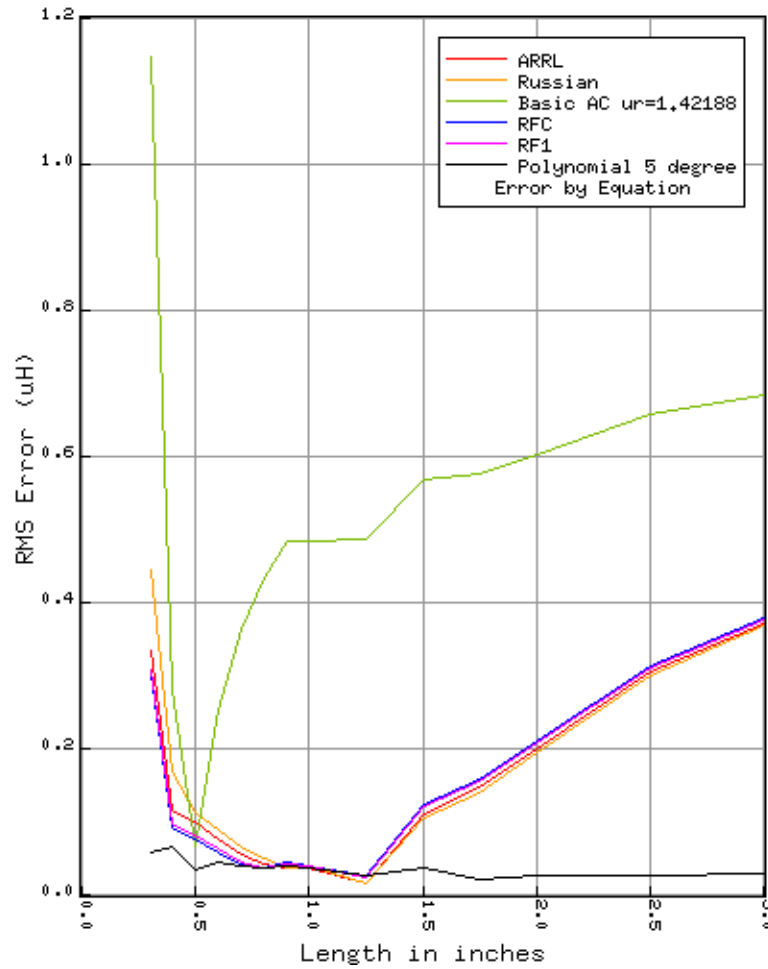


Figure 18: RMS error in length vs μH , 10 turns, 1" diameter, #22 wire

As before, 30 measurements were taken of each coil spread over a few weeks. The RMS error is computed from each measurement, not the mean.

8 Coil Radius

Next, we try the same experiment, keeping the length and turns the same, but increasing the diameter. The wall thickness is 0.125", length 1" and 10 turns. I ran both PLA and the

iron composite PLA as shown in Figure 19.



Figure 19: 10 turns, .1 spacing, diameter 0.5 to 1.75", #22 wire. Plain PLA front, Iron Composite in the back.

Performing the 30 measurements and computing the mean for both plain and composite PLA shown in Figure 20. After numerous attempts it appears that the 1.5" plain PLA diameter coil has anomalous values - perhaps one too many turns and has been removed. See *pladia.csv* on page 75.

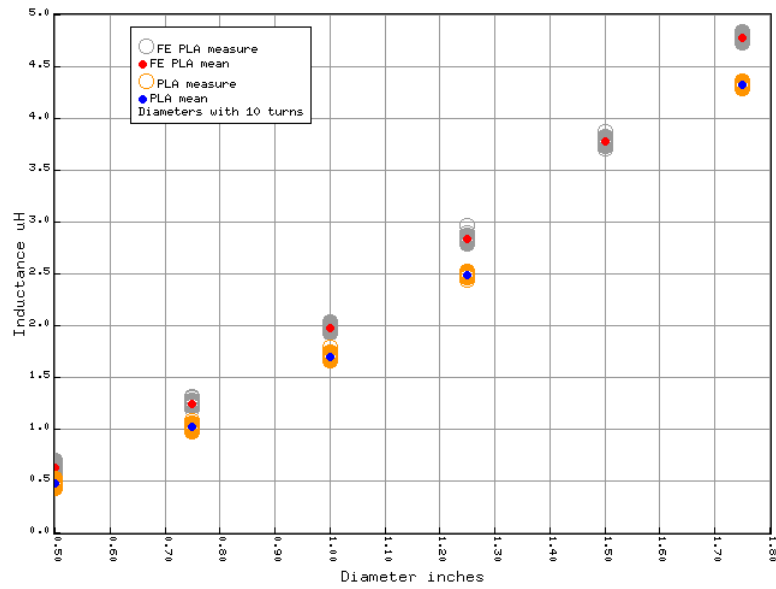


Figure 20: Diameter vs inductance

Next we try the same experiment to check predictions, first with plain PLA. We compare the measured inductance (30 times) against that predicted keeping track of the RMS error and plotting it. Except for an aspect ratio of 1.0, most of the equations perform really poorly. The RF1 equations 5-6 perform nearly as well as the polynomial fit.

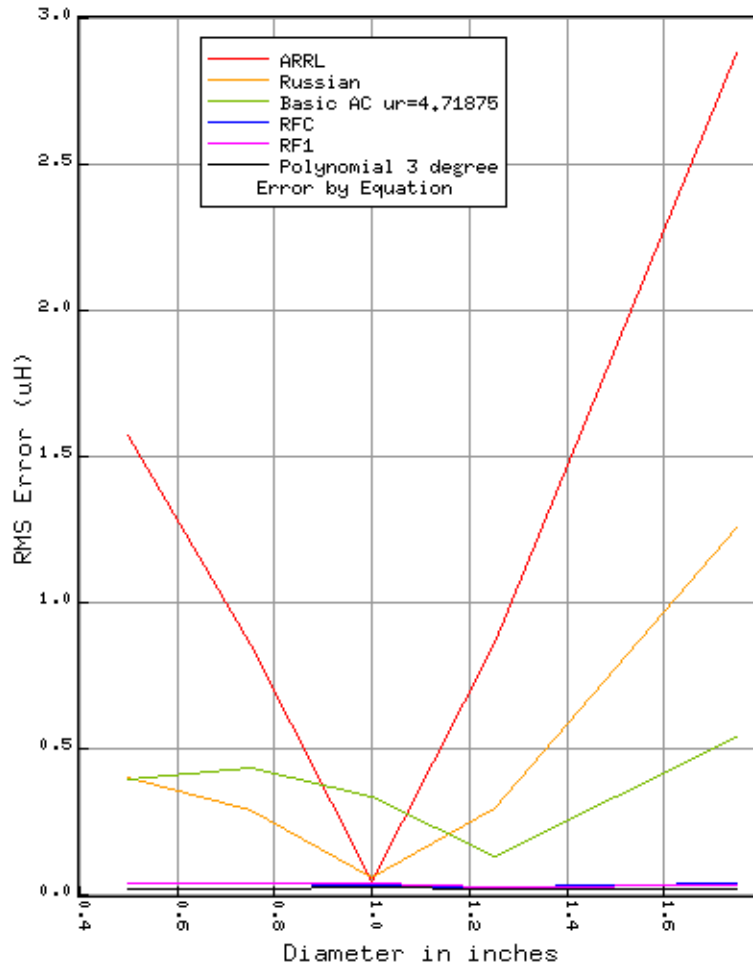


Figure 21: Really bad results except for aspect ratio of 1.0

Note that the μ_r value is ≈ 4.7 as opposed to ≈ 1.4 for the length experiment provided that equation 4 is correct, which is doubtful. The 3rd order polynomial as a function of diameter in inches is:

$$L = -0.155054 + 0.574652d + 1.47545d^2 - 0.195223d^3 \quad (29)$$

For the iron composite PLA, the results are considerably different. Again, the ARRL equations fit poorly, but the others not horrible. The 4th degree polynomial has the best performance over this set of aspect ratios. See *fedia.csv* on page 22.

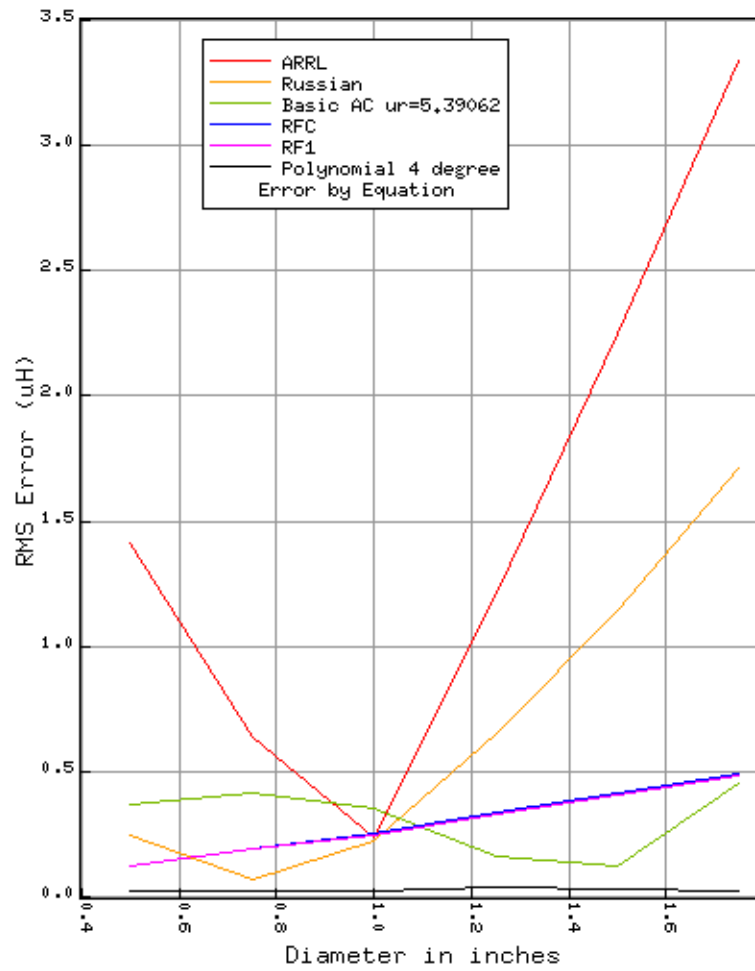


Figure 22: Iron composite diameter vs RMS error.

$$L = -0.206687 + 1.21172d + 0.836489d^2 + 0.249323d^3 - 0.110222d^4 \quad (30)$$

9 Wire Gauge

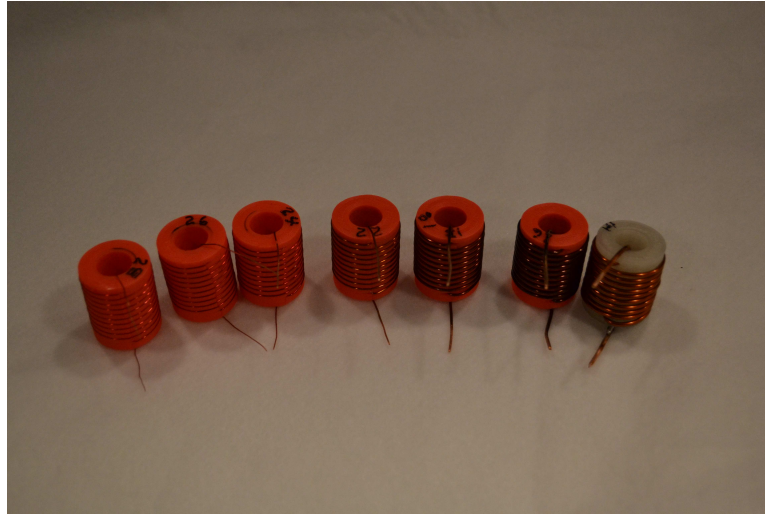


Figure 23: 10 turns, .1" spacing, wire gauges 14 to 28.

None of the equations accounts for wire size. What happens if we wind 10 turns on a 1" diameter PLA 50% filled, 0.125" wall 1" in length and vary the size? Unexpectedly, the inductance of these coils increases with the increasing wire gauge. For the wire gauges I had on hand, graph 24 show increasing inductance from #14 gauge to #28 gauge (missing #20 gauge). Winding #14 on a small coil requires strong fingers and tightening it caused a wire break (fixed by soldering a new lead).

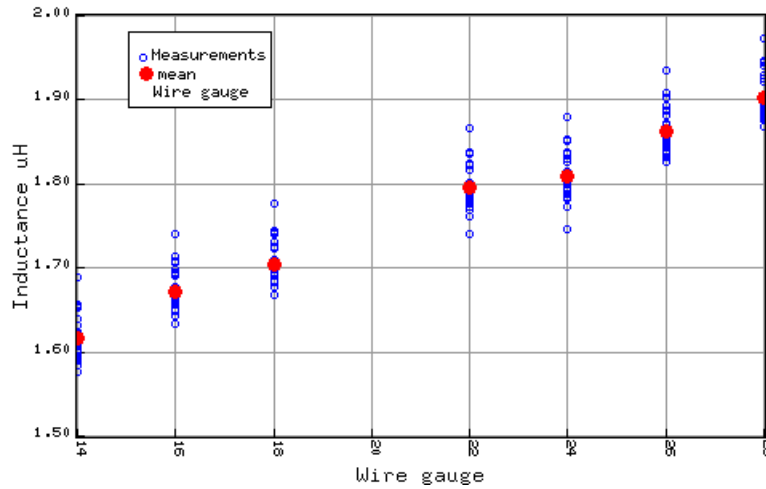


Figure 24: Wire gauge vs inductance.

Some caveats. When printing for finer gauges, we're up against the resolution of the 3D printer and some of the inductance may be a function of the wire center not being exactly at the 1" diameter. Likewise, the wire indents are made up of 20 short straight cylinders for each turn. Though these are lined up, the wind is not completely circular or even worse the channel may not extend far enough into the main cylinder.

Fitting curves to the values shows a straight line to have the best RMS error for this data set. For wire gauge g , 10 turns, 1" diameter, 0.25" wall thickness, plain PLA, the inductance the larger the gauge, the smaller the inductance.

$$L = 1.34679 + 0.0198398g \quad (31)$$

To test this, I printed a 3" 10 turn coil but with #14 gauge wire with the same 10 turns used for the #22 gauge form as shown in Figure 25. The theory being that changing the wire gauge from #22 to a smaller gauge will increase the inductance and a large gauge decrease the inductance by a linear amount.

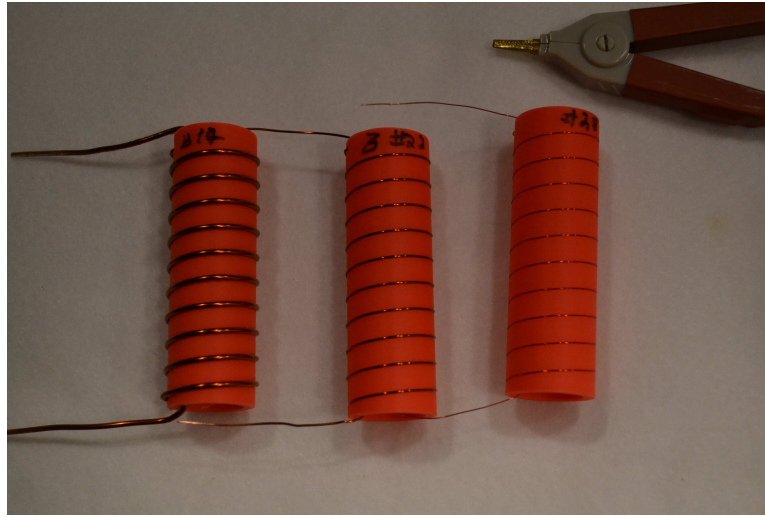


Figure 25: #14, #22, and #28 gauge 3" 10 turn form.

The inductance of the 14 gauge 1" coil is $\approx 1.62\mu H$, the inductance for #22 is $\approx 1.8\mu H$ a ratio of 0.9. The #28 gauge 3" coil ratio is about 1.1

9.1 Extending the Radius, Length, Turns Manifold

To extend the prediction space I created some additional coil sequences. All were wound with #22 wire,

Length 1", 1.25", and 1.75" with 5, 10, 15, 20, and 25 turns. For 1.75" there were additional 30 and 40 turn coils.

Diameter 0.375" (thinner walls), 0.5", 1.5", and 2" with 5, 10, 15, 20, 25, and 30 turns.

The results in Figure 26 show an expected increase with the square of number of turns.

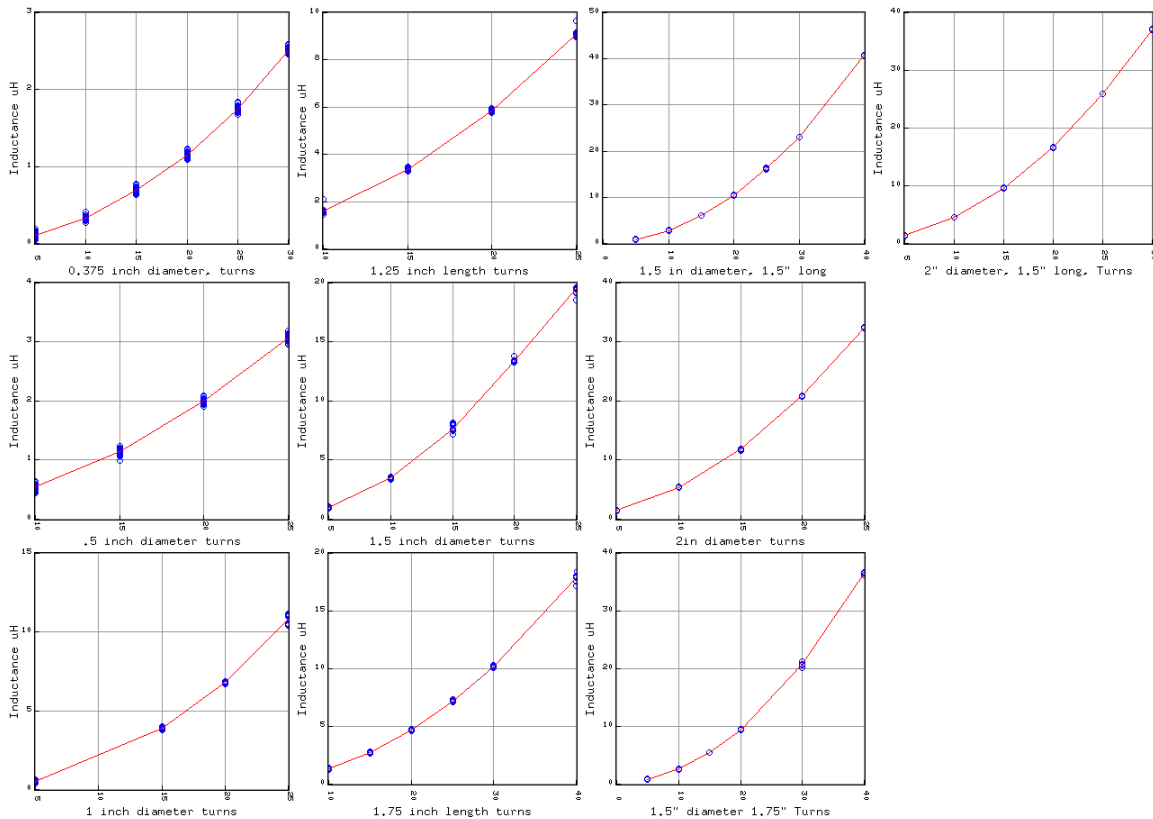


Figure 26: Measurement of different lengths, diameters, and turns.

There were 53 coils with 30 measurements for a total of 1590. This greatly helped the final polynomial fitting. ‘

10 Dependent Measurements

In the previous sections we varied only one quantity per experiment. But a typical coil has the number of turns and length in a linear relationship. For #22 gauge wire and 3D printing I used:

$$l = 0.028N \tag{32}$$

This works for #22 gauge wire with a .001" spacing between turns, mostly to account for slop in the 3D printing. To test this idea, I acquired a new 3D printer with higher resolution, devoted more compute time to higher resolution STL files and printed 78 more small coils

with 5, 6, 7, 8, 9, 10, 12, 14, 18, 24, 32, and 40 turns with radii of 0.125, 0.1875, 0.25, 0.3128, 0.375, and 0.4375.

The coils are not perfectly round and the channels are near the printer resolution. I measured around a representative coil sample at 45 degree intervals and computed the mean with the results in Figure 10. The real diameter is about 1.028 times the printed diameter. The diameters are the mean to the wire center.

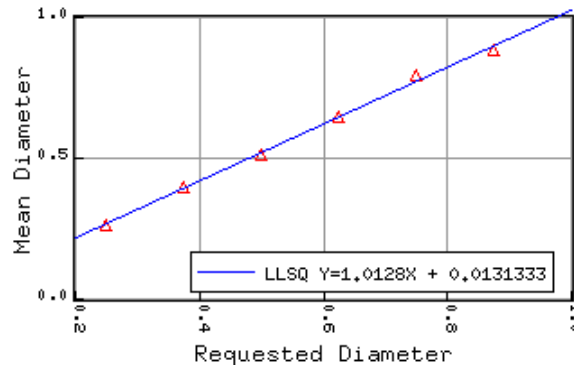


Figure 27: Print diameter, measured mean diameter all in inches.

The wire channels are a series of small cylinders removed from the overall cylinder. By default these are every 20° which may be sufficient for small diameters but has exaggerated errors on larger ones. The larger diameter coils were printed every 5°. Unfortunately, the extra cylinders caused openSCAD computational grief - some conversions took over 5 hours on a fast machine. In the Figure 28 we see the difference between low and high resolution for a 10 turn coil, 1" in diameter. The red helix varies considerably in depth and radius. The blue helix took nearly 14 hours to convert to STL.

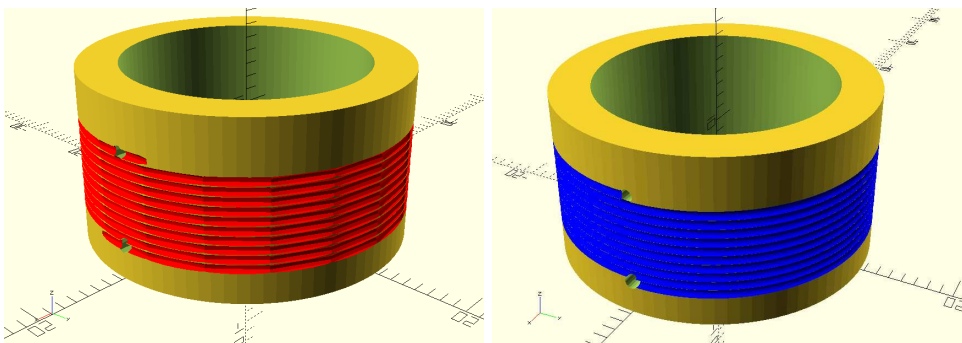


Figure 28: 18 cylinders per turn, 180 cylinders per turn.

These two forms were measured a number of times with inductances around $3.2\mu H$. The means were separated by 9.2 nano-Henrys with a zTest significance of 0.089 so in all likelihood they are the same.

After measuring the coils of different diameters and turns, I compared the measured mean inductance for each coil against the ARRL equation 1 and the multivariate polynomial from genetic algorithm approximation.

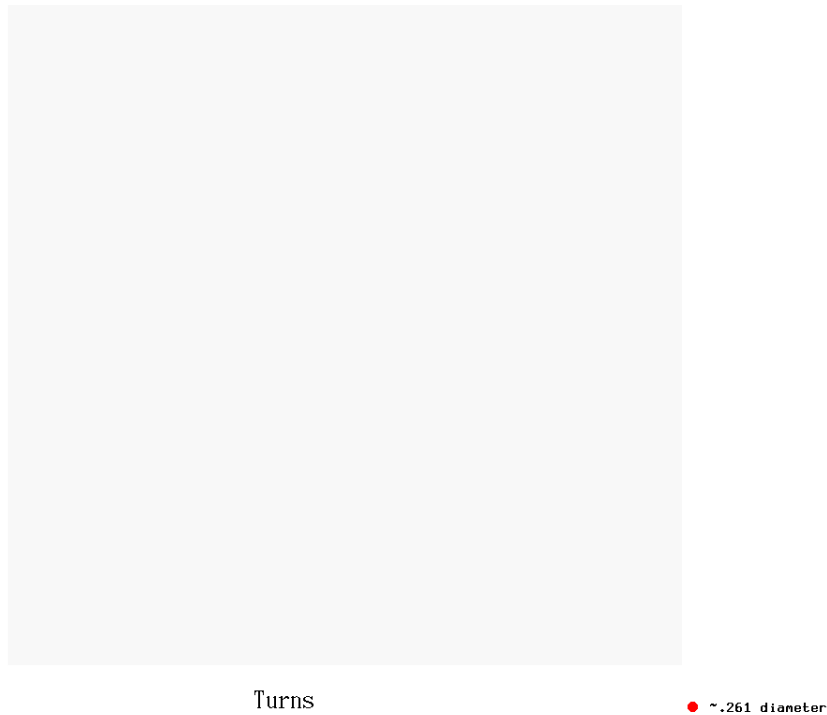


Figure 29: Measured vs ARRL equation and KI7NNP prediction.

The equation used for the KI7NNP prediction is:

$$\mathcal{L} = \mu_0 \frac{(0.005 - 2.4r - 761.51r^2 + 1000r^3)(0.395 + 0.145t - 0.47t^2)}{11.48 + 202.975r + 1000r^2 + 1352.1l + 446.135l^2} \quad (33)$$

where r and l are in meters and the result in Henries. I also note that:

$$l = 0.0007122t \quad (34)$$

The results show a much better prediction than formula 1 provided that:

1. #22 gauge wire,

2. 0.028" spacing between turns,
3. $0.1\mu H < \mathcal{L} < 20\mu H$
4. Diameter between .261" and 0.882".
5. $\mu_r = 1$

Going beyond these limitations will result in unpredictable errors.

The results were somewhat disheartening with the following identified problems:

1. Two different PLA types behaved differently in the printer with shrinkage slightly different.
2. Some of the coil walls were too thin resulting in slightly oval winding's when tensioned.
3. The separation between winding's varies somewhat caused by limited printer resolution (less than 0.001" but still there).
4. The LCD meter has greatly diminished accuracy below 0.5μ . Fitting the polynomials to the bad data is unwise and makes the fit to more accurate measures less reliable. To some extent, this avoids fitting to the vicissitudes of one single LCR meter.
5. Shrinkage after printing is irregular in all 3 axes for this printer and depends somewhat on the PLA manufacturer and geometry.

To solve some of the problems I measured this experiment's coils at 45 degree increments near their center, subtracted one wire diameter (0.027"), rebuilt the database, removed some of the coils at the margins of the LCR sensitivity and reran the equation fitting.

11 3D Printing Problems

The previous experiment demonstrated that both shrinkage and X/Y irregularities were common. To test this hypothesis, I printed a number of 1" cubes with 2.85 mm PLA from different manufacturers and different colors. The fill was set to 100% and resolution to the highest for a .5mm nozzle.

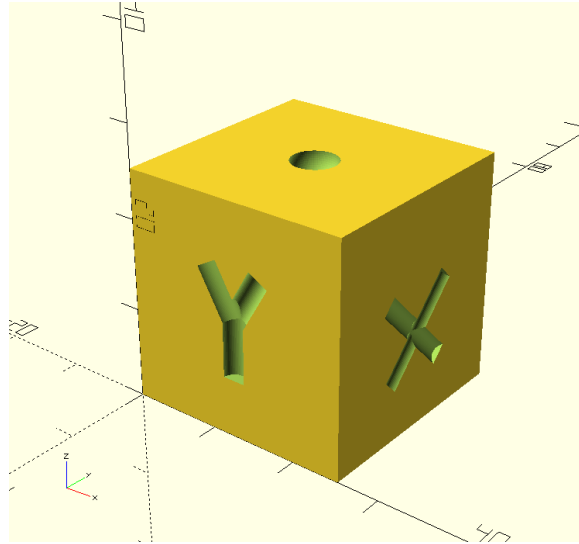


Figure 30: Cube with side markers. Design 1" x 1" x 1"

Filament	Fill	X	Y	Z
PolymakerLite lime green	100	0.998	0.994	0.995
	20	0.998	0.995	1.005
PolymakerLite white	100	0.998	0.995	0.992
	20	1.000	0.995	0.992
	highres	1.001	0.997	1.009
Unknown clear	100	1.000	0.997	0.988
	20	0.995	0.994	0.965
MH BuildSeries Forest Green	100	1.007	1.007	.980
	20	.998	1.001	0.994

Table 2: Cube measurements at top/center

I also printed plain cylinders to measure length and diameter using a test stand and a dial indicator to measure every 5 degrees around the circumference. For a 1" diameter cylinder printed at 2 different resolutions the results are shown with the cylinder deviation 10 times normal.

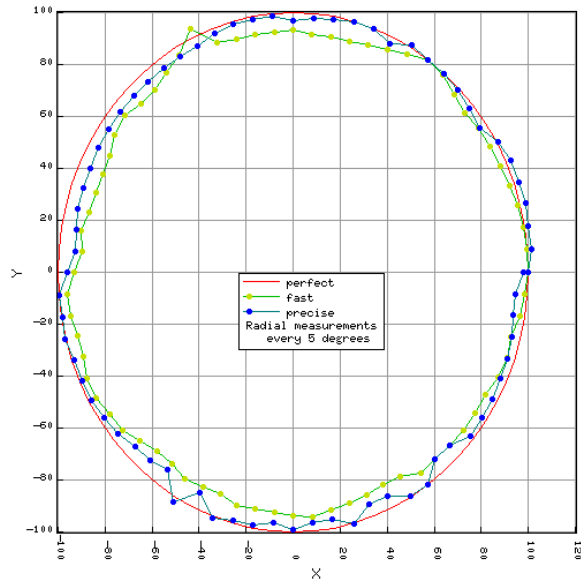


Figure 31: 1" cylinder at 5 degree increments.

I measured the lengths vs the predicted length on a number of coils and found that the length was accurate to 0.001" in all cases. However, as shown in Table 2, the total Z varies considerably. I expect this has to do with the Z axis adjustment and the first few layers. It also appears that filament from the printer manufacturer performs better than generic. If you're used to the .001" accuracy of a milling machine, don't expect such from retail additive printing.

12 LCR Meters

Earlier measurements were made with the East Tester ET4410. As a check I borrowed a hand held Keysight U1733C LCR meter and later purchased a bottom of the line CAMWAY BM4070.

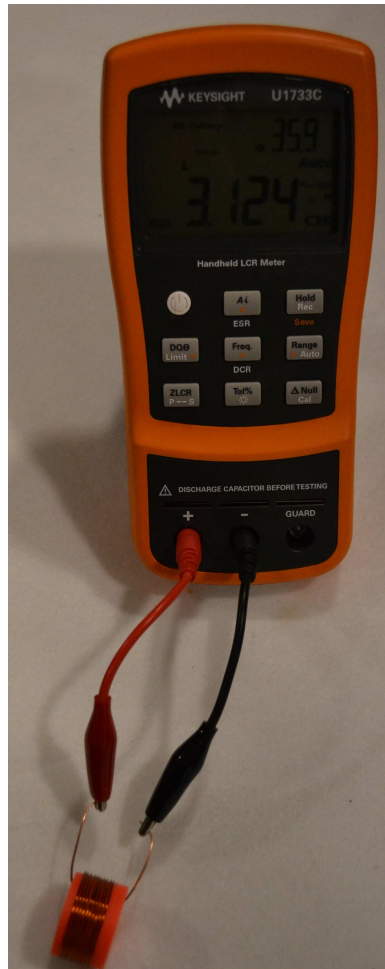


Figure 32: Keysight U1733C LCR meter

I repeated the measurements on the 10 turn coils with lengths varying from 0.3" to 3" (Figure 16) and recorded the set 30 times. Comparing the mean of each set of 30 measurements shows that the Keysight averages some 4% higher than the East Tester.

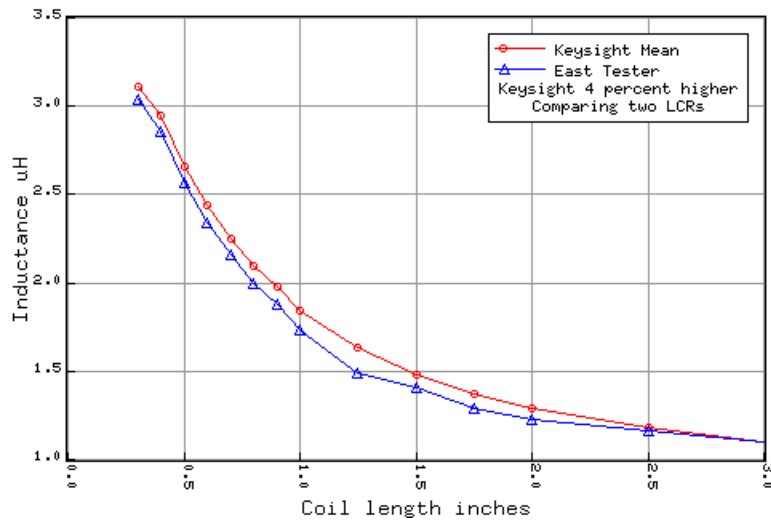


Figure 33: Varying coil length, East Tester vs Keysight LCRs

It appears that the Keysight measurement is more stable than the East Tester. Figure 34 shows the median values of each series of tests, the minimum and maximum, and the standard deviation. The mean of the standard deviations is about 6.4 nanoHenries.

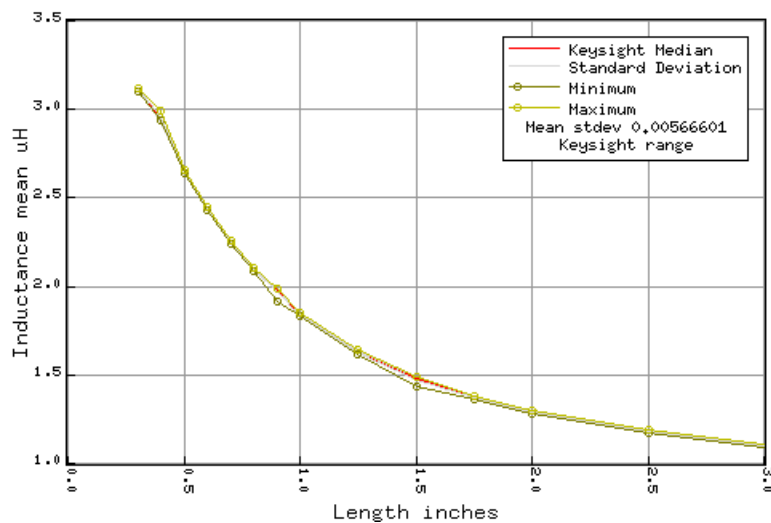


Figure 34: Varying coil length, Keysight variability each size.

On the other hand, the East Tester, in Figure 35, has a standard deviation of 32.5 nanoHenries, about 5 times worse.

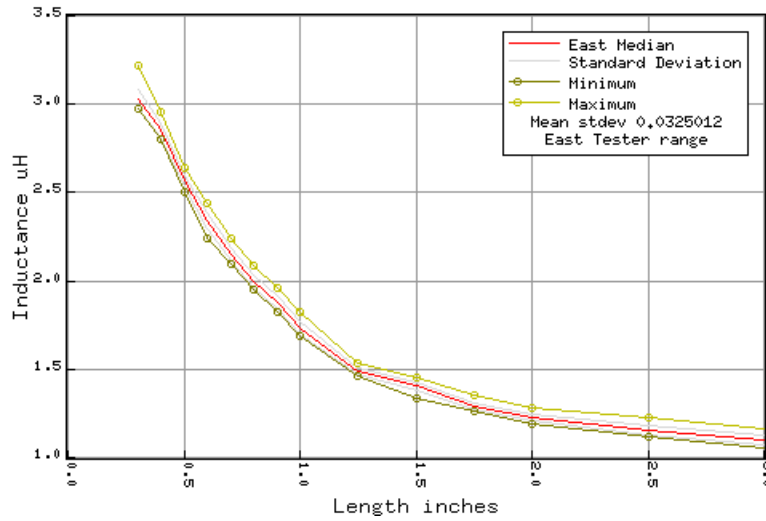


Figure 35: Varying coil length, East Tester variability each size.

I wondered if the ranges returned for each inductor varied with inductance. A wide range would indicate either insufficient calibration, uncompensated temperature variations, poor connections or varying parasitics from the orientation or connection length. In Figure 36 it's clear the East Tester has greater variability as the inductance goes up - perhaps a sign of insufficient digital capture, but the Keysight goes down somewhat but for unknown reasons. Perhaps the inductance range isn't sufficient to draw conclusions.

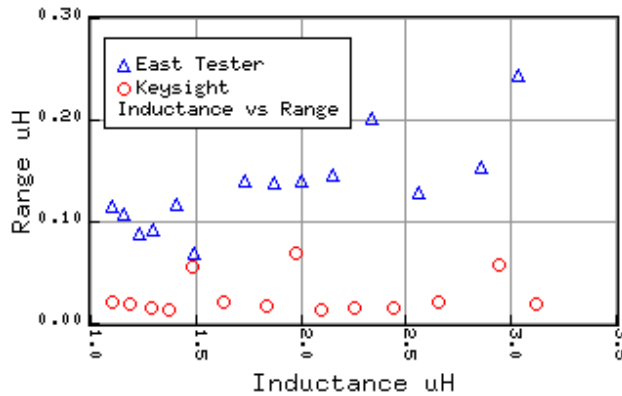


Figure 36: Range per inductance value for two testers

The CAMWAY LCR has good agreement with both the East Tester and Keysight but much less resolution. I measured the same coils as above with calibration between each

measurement plotted in Figure 37. Like the Keysight, the CAMWAY measured slightly above the values shown in green multiplied by 10.

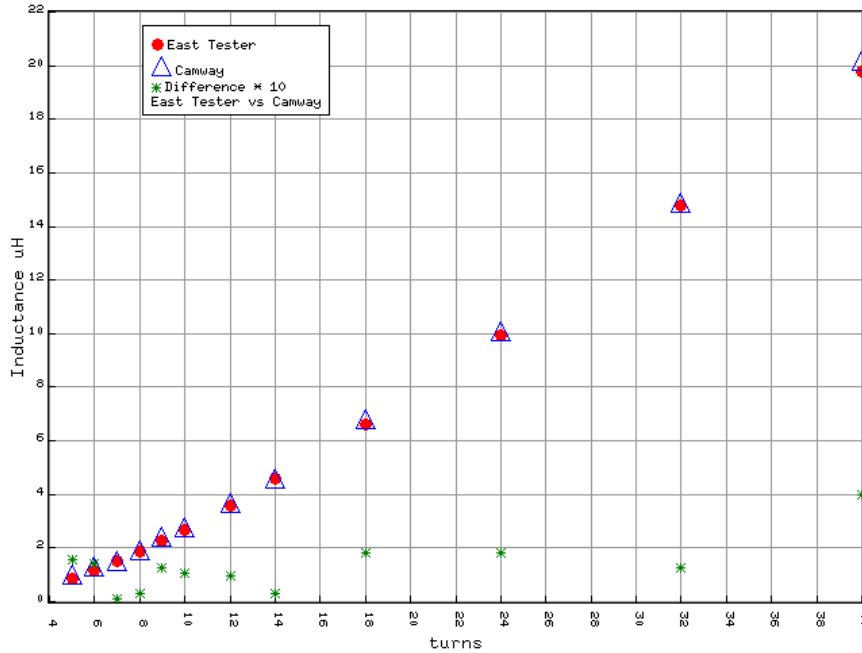


Figure 37: Comparing CAMWAY BM4070 to East Tester 4410

This is insufficient evidence to convict any instrument reviewed. Just because one has more stable readings than the other, doesn't make it more accurate. After all, a clock that's stopped is accurate at least once a day. Potential causes:

1. **Probes** The East Tester probes are Kelvin whereas the Keysight probes are short lead alligator clips. The East Tester probes are gold plated but do not have sharp teeth. They are spring loaded but not strong. The Keysight alligator clips are chrome plated, sharp and have a very strong spring, enough to make your hand cramp when testing a number of coils. The CAMWAY probes are similar but not as strong.
2. **Drift** Per its instructions, the lines powered East Tester is turned on once in the morning, wait 30 minutes before a test and then the calibration routine run. The Keysight tester is battery powered, the calibration period is short and is turned off between tests. The East Tester appears to drift during the day though in no recognizable pattern even when calibrated between runs. The CAMWAY tester drifts with battery discharge. It is best to calibrate between every measurement. Just remember to subtract the calibration value from the final reading.

13 Permittivity

As the PLA used is an insulator and most inductor calculations are based on air or iron cores. The parasitic capacitance associated with PLA becomes particularly important at higher frequencies. I measured the permittivity of PLA blocks of various fill percentages. The test stand is two 4"x4"x0.25" aluminum plates nestled in a PLA test stand separated by a 0.25" air gap.



Figure 38: Permittivity test stand.

A number of 0.25" panels were printed with varying degrees of fill from 3% to 100%. The capacitance values were then compared to the air gap capacitance of some $\approx 17.5\text{pF}$. The measurements were taken with a 100 kHz excitation frequency with 2 volts output.



Figure 39: Permittivity test panels.

The permittivity does go up (PLA is more of an insulator than air) when measured against fill percentage. The PLA fills half the air gap between coils - used to keep them from slipping around. The odd colors come from using up various PLA spools with only short lengths left.

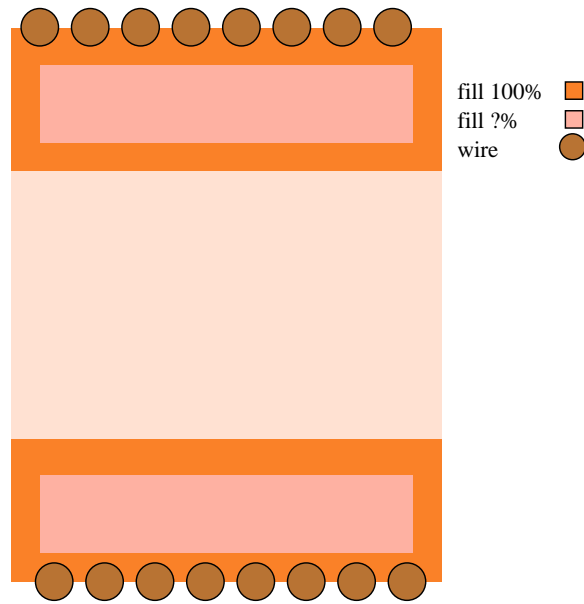


Figure 40: PLA coil forms with solid outside, inner fill.

The weight per panel vs fill percentage is not linear.

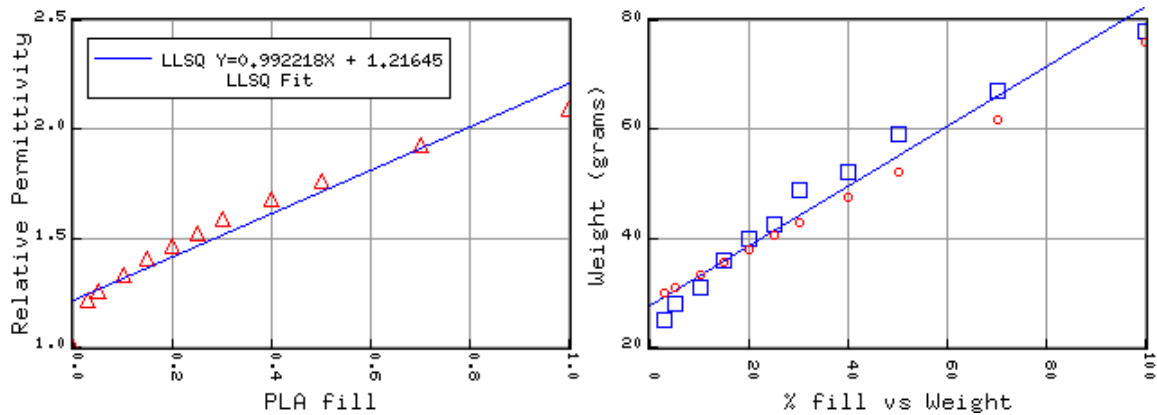


Figure 41: Capacitance vs fill percentage and weight (in grams).

However, from the previous demonstrations, the value of μ_r is not much affected by the fill PLA percentage unless the material is the iron composite type. However, given that dry PLA is a better insulator than air, the parasitic capacitance of a coil may be greater thus causing problems at frequencies higher than the 100 kHz of the measurements.

14 Q Measurement

A coil's "quality" measurement is:

$$Q = \frac{2\pi fL}{r} \quad (35)$$

where r is the coil's DC resistance and f the frequency measured at (until near the self resonant frequency). I measured some coil Q's with the East Tester, but the results tend to wander considerably so values with more resolution than integers are suspect. The resistance of most coils of #22 wire is quite small. One foot of #22 has a DC resistance of .0164 ohms so any finger grunge or bad contact can have a great variation on the measured Q. SRF measurements (see Section 15 on Page 50) show near 40 MHz and our excitation frequency is 100 kHz so we're not in the area where this should cause problems with skin effect and so on.

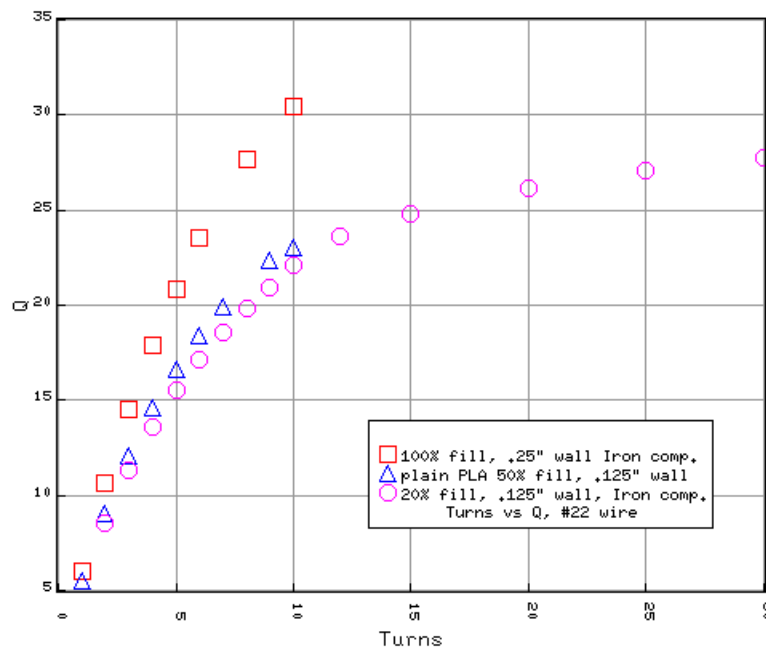


Figure 42: Q of plain PLA, .125" wall and .25" iron composite PLA

Note that the X axis in Figure 42 is number of turns, not inductance. But I'm still unable to explain why the plain PLA cores have a slightly higher Q than the 20% iron composite ones.

15 Self Resonant Frequency

As the permittivity of PLA is better than air, we would expect the self resonant frequency (SRF) do decrease over air. To test the SRF, each coil was hooked to a calibrated nanoVNA and we look for the first 180 degree phase change. The sample size was set to 501, and the range adjusted so that the change is in the approximate middle of a 1 MHz sweep.

Most of this capacitance is between the coil winding's which are half air and half PLA. I first tested the SFR of the variable length 10 turn coils shown in Figure 43. Here there are 10 turns of #22 on 1" diameter. As shown in Figure 16, the SFR increases as the distance between turns expands indicating less capacitance.

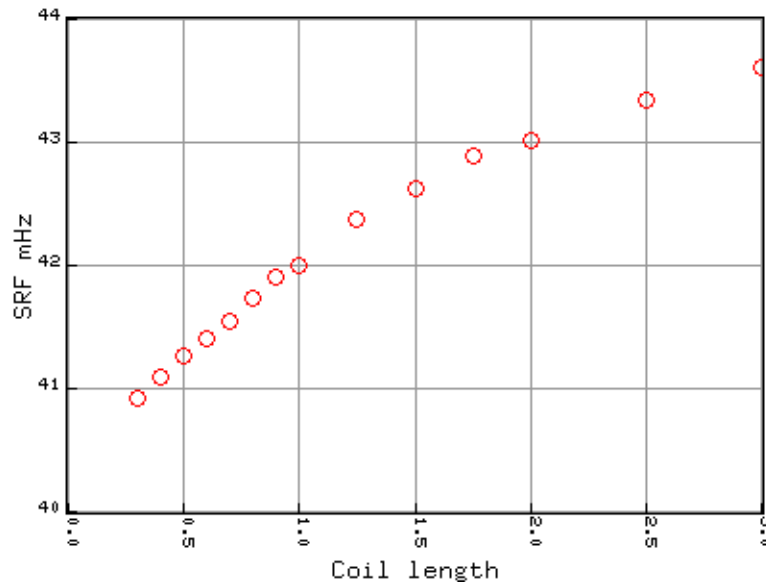


Figure 43: Coil length vs self resonant frequency

The type of core matters as well. In Figure 44 Are measurements of three coil types. The single air core coil serves as a baseline at 41.717 mHz (shown at fill=50%). The iron composite PLA is fairly close to this value though somewhat lower as it is not very conductive. The conductive PLA fills half the space between turns making a near multi-plate capacitor thus reducing the SRF considerably. The effect increases somewhat with the fill density. The variability between 5% and 10% fill is difficult to explain but may have to do with the internal orientation of the “star” fill pattern. Two identical prints have nearly the same SRF and inductance.

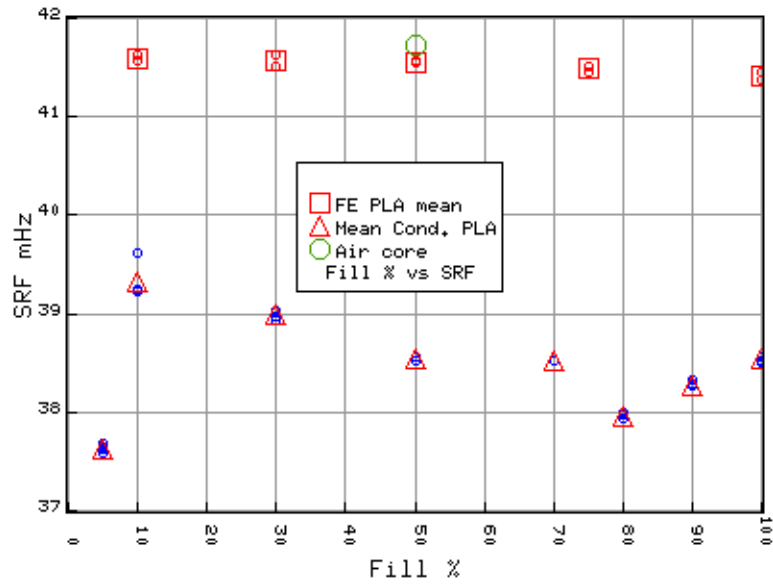


Figure 44: Coil core type and fill percentage vs SRF.

16 Repeatability

I printed 8 coils with the same dimensions, wire, and number of turns. 1" diameter, 0.5 inch coil with 10 turns of #22 wire.



Figure 45: 8 coils, all the same.

These are labeled A-H and should measure around $2.6\mu H$.

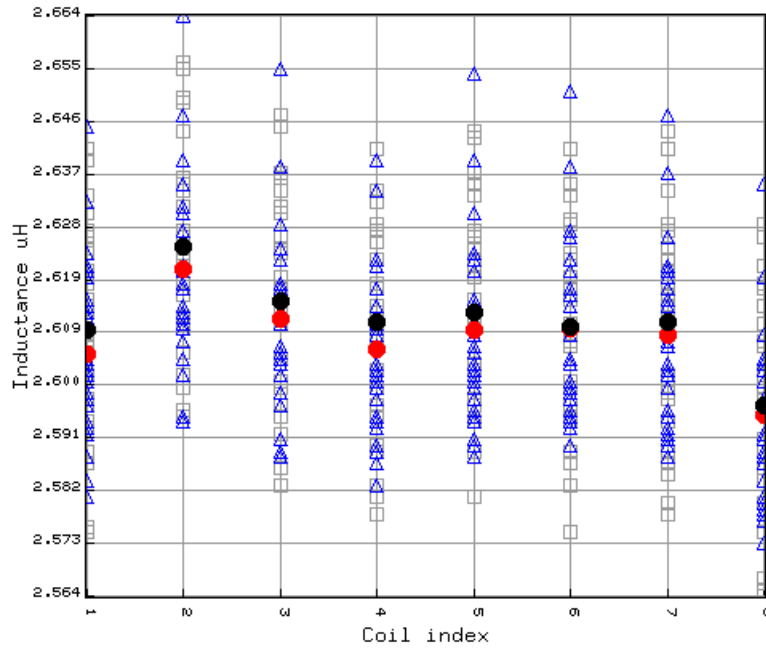


Figure 46: Measures of 8 identical coils, dry and humid

The mean value for raw data for 30 measurements of each coil is $2.602\mu H$, with a standard deviation of $0.0132\mu H$ a variation of about .5%. Removing any values with a z-Score greater than 1.25 reduces the mean to $2.601\mu H$ with a standard deviation of 0.0127 or variability of about 0.41%.

The maximum skew value was 1.52 indicating that the measurements do not have a normal distribution around the mean - they tend to have a long tail in the positive direction. This may be a function of the meter. The kurtosis values show that the 30 measures are reasonably normally distributed (Table 3).⁵

⁵Tables 3 and 5.

	mean	stDev	skew	kurtosis
1	2.60573	0.01399	0.737176	3.38026
2	2.62037	0.0152598	0.746144	3.46321
3	2.61173	0.0150171	0.805701	3.43339
4	2.60657	0.013645	0.535068	2.6187
5	2.6098	0.0152844	0.977893	3.46694
6	2.61007	0.0147366	0.87618	3.06223
7	2.60903	0.0146252	0.63661	2.73851
8	2.59517	0.0142395	0.696502	3.134

Table 3: Same coil statistics, 30 measurements.

It should be apparent from Figure 46 that coil's 2 and 8 are problematic for some unknown reason. The zTest Table 4 bears this out. Here comparisons that are at least marginally different are in bold.

	1	2	3	4	5	6	7	8
1	*	-3.872	-1.601	-0.234	-1.075	-1.168	-0.893	2.899
2	3.872	*	2.209	3.692	2.680	2.659	2.937	6.613
3	1.601	-2.209	*	1.395	0.494	0.434	0.705	4.385
4	0.234	-3.692	-1.395	*	-0.864	-0.955	-0.675	3.166
5	1.075	-2.680	-0.494	0.864	*	-0.069	0.199	3.837
6	1.168	-2.659	-0.434	0.955	0.069	*	0.273	3.983
7	0.893	-2.937	-0.705	0.675	-0.199	-0.273	*	3.721
8	-2.899	-6.613	-4.385	-3.166	-3.837	-3.983	-3.721	*

Table 4: zTest all 8 against each other

The discrepancies can be accounted for by the measured diameters of the wire at the coil center, i.e. the diameter outside the wire hence the larger value. Figure 47 shows the relation between mean diameter and inductance with coils 2 and 8 being the outliers.⁶

⁶Data found in *t10d.csv*, comparison program *t10same.ja*.

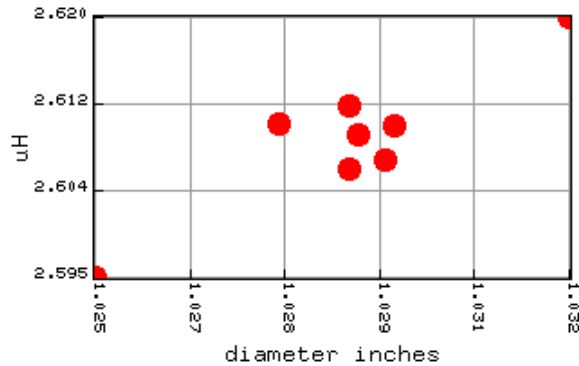


Figure 47: Diameter vs inductance, 8 coils the same.

I also examined coil diameter for those in Figure 45 as shown in the following table with diameters in inches. The coils were printed with each dimension multiplied by 1.02 to account for shrinkage. Measurement was at the 90 degree point from the wire exit and does not include the wire.

Coil	Diameter "	Humid "
A	0.997	1.000
B	1.000	1.002
C	0.999	1.000
D	1.000	1.002
E	1.000	1.000
F	1.000	1.001
G	0.997	1.001
H	0.996	1.001
Mean	0.9986	1.00087
Median	0.9995	1.001

Table 5: Caliper measured coil diameters for 1" nominal.

The average dimension is within about 0.14% of the specified diameter. However exposing the coils to 97% - 99% relative humidity for 1 day results in an average increase in diameter of about 0.25%.

17 Humidity

It's well known that a lot of printable plastics are hygroscopic (absorb moisture). I placed the coils in Table 5 in a tight plastic box with a bowl of water and a relative humidity sensor. I waited two days for the relative humidity to stabilize between 97% and 99% at around 65 F.

The mean of these measurements is the filled black circle in Figure 46. The measurements show that the inductance increases about 0.43% at high humidity, not bad but not horrible.



Figure 48: High humidity chamber.

A further round compared larger coils with a varying number of turns. The results in Figure 49 show the same effect.

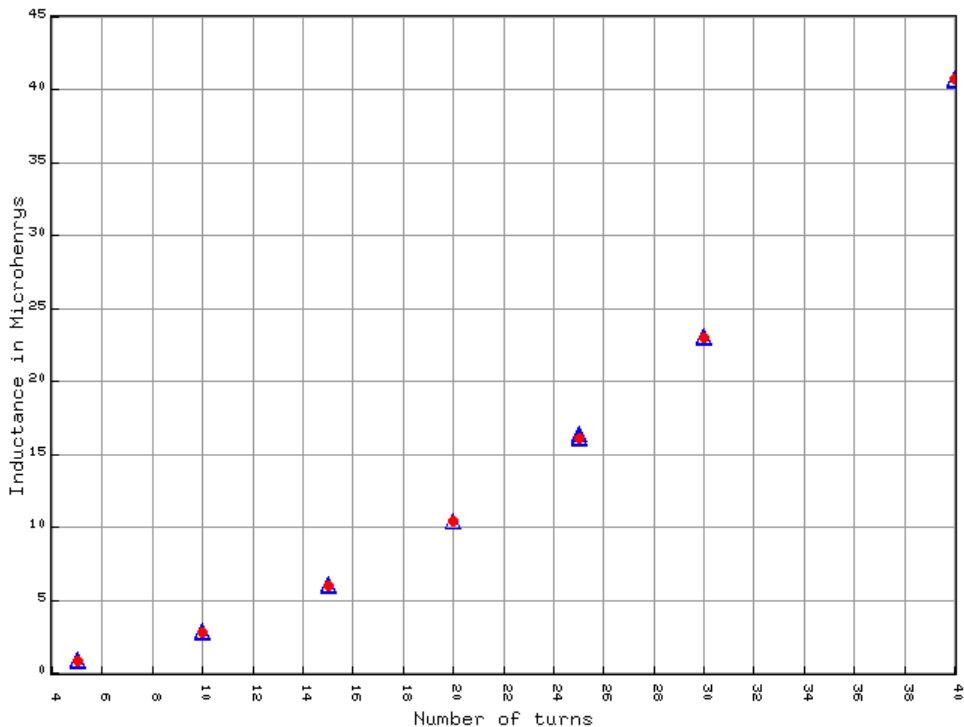


Figure 49: Inductance high vs low humidity, same coils.

Conclusion: humidity doesn't matter much.

18 Equations

Iterating through different equations is difficult. I tried fitting a few different equations with the genetic program **indequ** compiled for different versions. These were run any number of times to “completion” so results can be compared.

turntest.csv Fixed spacing, radius, #22 gauge wire, vary turns. 8 coils.

cspacing.csv #22 gauge coils, 0.028 spacing only, no inductance below $.5 \mu H$. 61 coils.

all22.csv All coils with #22 wire. No inductance below $.5 \mu H$. 149 coils.

all.csv All coils, different wire sizes, PLA cores only. 173 coils.

All equations have limitations. Table 6 gives the limits for equations by the file they were optimized against. Attempting a value beyond any of the limits is tempting fate.

Subset	Entries	Turns	Gauge	Diameter	Length	μH
<i>turntest.csv</i>	8	8-50	22	0.5"	.32" - 2"	0.612 - 7.06
<i>cspacing.csv</i>	61	5-40	22	.258" - .866"	.14" - 1.12"	.516 - 19.8
<i>all22.csv</i>	149	5-40	22	.261" - 2"	.14" - 3"	.502 - 40.73
<i>all.csv</i>	173	5-50	14-28	.261"- 2"	.14" - 3"	.502 - 40.73

Table 6: Optimization subset limits

The following equations use these variables.

R Coil radius in meters.

N Number of turns, an integer.

L Coil length in meters.

W Wire radius with insulation in meters.

\mathcal{L} Inductance in Henrys.

18.1 Tukey Plots

These are an attempt to show the errors within a restricted bin of values, usually $2 \mu H$. The rectangular box holds 50% of the error values, the bar across the middle is the median of all bin values. Above and below the box there may be a T (if there are more than 3 values). Values that lie within 1.5 times the box range lie within this area. If there are any values that fall outside this range, they appear as solid black circles. A plain circle indicates that only one value occurred in that bin.

18.2 ARRL Equation

Equation 1 on page 7 can be modified with different integer coefficients to better fit the data. Using an exhaustive search with limited integer values shows the limits of this equation. Repeats using the genetic algorithm resulted in different floating-point coefficients but with nearly identical results.

$$\mathcal{L} = \frac{Ad^2n^2}{Bd + Cl} \quad (36)$$

Subset	Original RMS	New RMS	A	B	C
<i>turntest.csv</i>	10.24%	1.82%	44	1283	1585
<i>cspacing.csv</i>	12.52%	5.54%	43	864	1909
<i>all22.csv</i>	13.06%	10.44%	11	229	456
<i>all.csv</i>	14.42%	11.88%	16	337	664

Table 7: Modified ARRL coefficients for different subsets

The ARRL equation is probably OK for general purposes but tends to show higher values than those read by LCR meters. For truly accurate values, it is insufficient.

18.3 Simple

For the #22 with fixed spacing coils I note that $l = 0.027N$ so remove length from the equation. This performs reasonably well but not killer. It appears to have some trouble at small values (perhaps the fault of the LCR meters) and consequently at the larger ones as well. In the code, this is called the SIMPLE model and was optimized against the *cspacing.csv* test file.

$$\mathcal{L} = \mu_0 \frac{(r_0 + r_1 R + r_2 R^2 + r_3 R^3)(t_0 + t_1 N + t_2 N^2)}{c_d} \quad (37)$$

Coefficient	Value
r_0	0.04577932701610
r_1	-3.47318092941792
r_2	-32586.59629999914978
r_3	990000.68900003179442
c_d	459987.89609983825358
t_0	81262.26570000193897
t_1	-30132.44869999826915
t_2	-1717.08300000000872
t_3	21.00967900000605

Table 8: SIMPLE coefficient values.

The best RMS error so far is 2.34% with the worst error 6.4%.

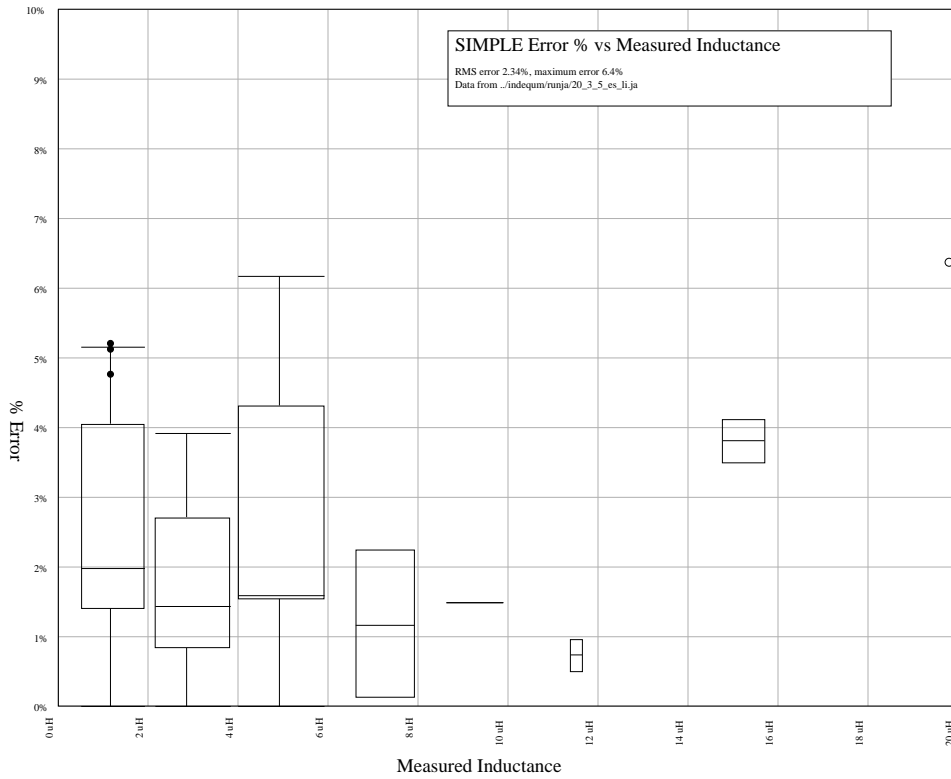


Figure 50: Simple Equation error bars (Tukey) optimized for *cspacing.csv*.

For the coils measured, the result is much more accurate than the ARRL equation.

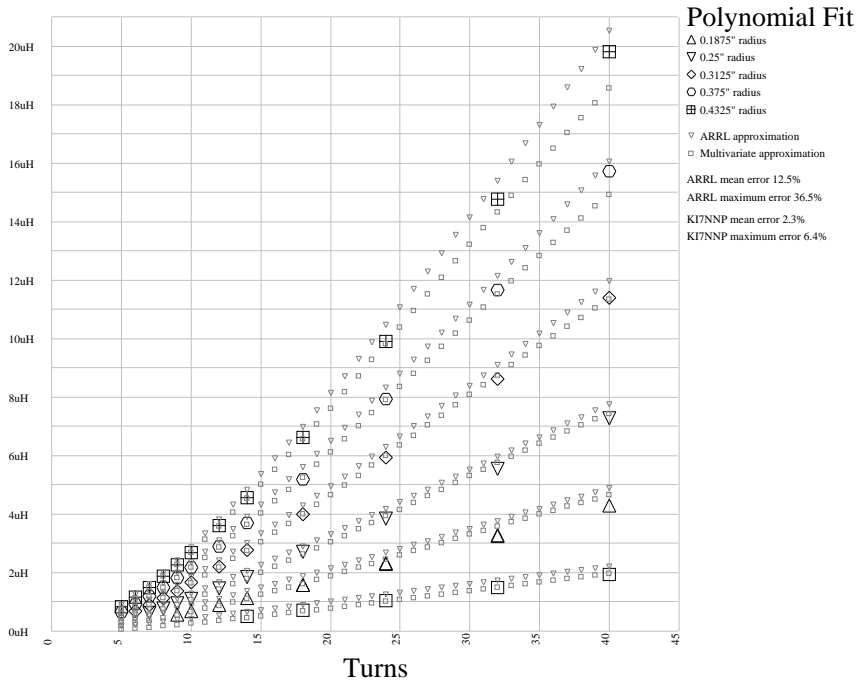


Figure 51: Simple Equation vs ARRL equation.

18.4 Equation 8

Mirrors ARRL formula but with extra powers on the terms. Optimized against all #22 gauge coils in *all22.csv* and called EQUATION8 in the code. The results are reasonable and can be used when the wire gauge is not important.

$$\mathcal{L} = \mu_0 \frac{(rn_0 + rn_1R + rn_2R^2 + rn_3R^3)(t_0 + t_1N + t_2N^2)}{(rd_0 + rd_1R + rd_2R^2) + (l_1L + l_2L^2 + l_3L^3)} \quad (38)$$

Coefficient	Value
<i>rd</i> ₀	-400.31259076938949
<i>rd</i> ₁	-47084.04000000069937
<i>rd</i> ₂	-988254.42500002332963
<i>l</i> ₁	-72990.26241361664142
<i>l</i> ₂	215265.18499991873978
<i>l</i> ₃	-1299810.50000000000000
<i>rn</i> ₀	0.00552478988815
<i>rn</i> ₁	-0.65814076724923
<i>rn</i> ₂	-37580.18161940149002
<i>rn</i> ₃	-182142.72499997518025
<i>t</i> ₀	-19.30499999999984
<i>t</i> ₁	12.08749991044770
<i>t</i> ₂	5.35551893791146

Table 9: Equation 8 coefficient values.

The results for the *cspacing.csv* being worse than all the #22 gauge coils indicating some attention made to the many more coils in the bigger set.

Subset	RMS Error
turntest.csv	6.14%
cspacing.csv	6.11%
all22.csv	3.90%
all.csv	6.11%

Table 10: Equation EQUATION8 Results

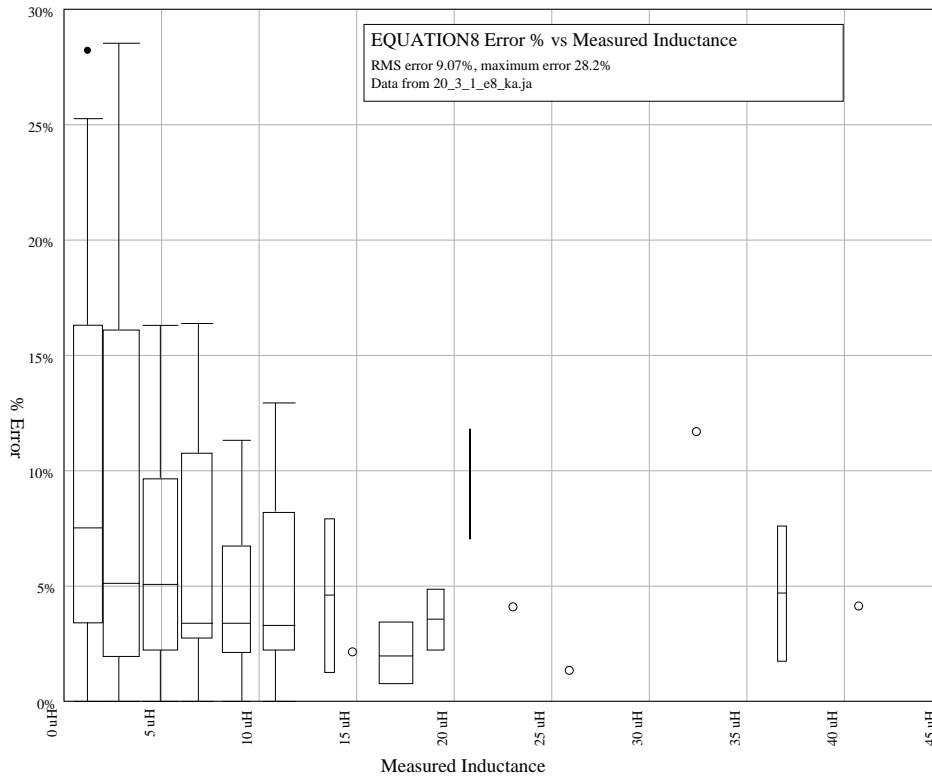


Figure 52: Equation 8 error bars (Tukey).

18.5 Equation A

Not a good idea as 5 degrees tends to fit well but will have many oscillations between measurements that don't bode well for predictions inside the measured range. Because the wire gauge is not included, we run this only against #22 gauge coils. Called EQUATIONA in the code.

$$\mathcal{L} = \mu_0 \frac{(r_0 + r_1 R + r_2 R^2 + r_3 R^3 + r_4 R^4 + r_5 R^5)(t_0 + t_1 N + t_2 N^2)}{l_0 + l_1 L + l_2 L^2 + l_3 L^3 + l_4 L^4 + l_5 L^5} \quad (39)$$

Table 11: Equation EQUATIONA Results

Coefficient	Value
r_0	-0.460000000000000
r_1	182.99999999999596
r_2	0.000000000000000
r_3	226674.20000000012806
r_4	100000.000000000000000
r_5	50047.399999999787688
t_0	3.720000000000000
t_1	-1.880000000000000
t_2	-0.520000000000000
l_0	-450.99999999999881
l_1	-24924.80000000019209
l_2	-0.200000000000000
l_3	0.600000000000000
l_4	19.600000000000000
l_5	-20.200000000000001

Table 12: Equation A coefficient values from 20_3_1_ea_ub.txt.

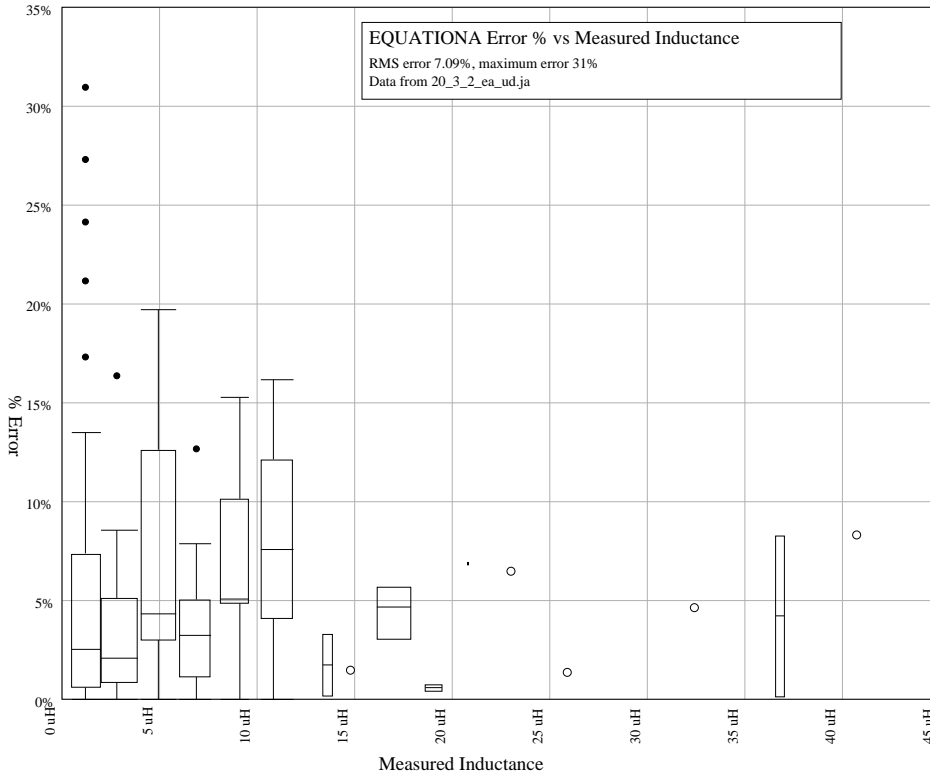


Figure 53: Equation A error bars (Tukey).

18.6 Equation C

Add wire gauge to Equation A. Called EQUATIONC in the code.

$$\mathcal{L} = \mu_0 \frac{(r_0 + r_1R + r_2R^2 + r_3R^3 + r_4R^4 + r_5R^5)(t_0 + t_1N + t_2N^2)(w_0 + w_1W + w_2W^2)}{l_0 + l_1L + l_2L^2 + l_3L^3 + l_4L^4 + l_5L^5} \quad (40)$$

18.7 Equation D

This accounts for wire size and variable spacing but removes the quintic on the radius and length of Equation A. Run against all coils except those with less than .5 μH . In the code, this is called EQUATIOND.

$$\mathcal{L} = \mu_0 \frac{(r_0 + r_1R + r_2R^2 + r_3R^3)(t_0 + t_1N + t_2N^2)(w_0 + w_1W + w_2W^2)}{l_0 + l_1L + l_2L^2 + l_3L^3} \quad (41)$$

alljan24.csv			
varsd.indequ			
Results	Mean	Worst	Seed
20_3_3_ed_cj.txt	3.99%	26.2%	987654321

Table 13: Equation EQUATIOND results

Coefficient	Value
r_0	0.01794
r_1	-8.31722000000232
r_2	-804.89380000006770
r_3	-0.0379
t_0	-5.25855999999957
t_1	1.52247999999994
t_2	0.73276000000004
w_0	-1.03136000000004
w_1	1.54125999999996
w_2	88.01792000000133
l_0	3.55340000000013
l_1	425.52679999991551
l_2	-1036.50319999997032
l_3	-18481.68420000031620

Table 14: Equation D coefficient values from 20_3_3_ed_cj.txt.

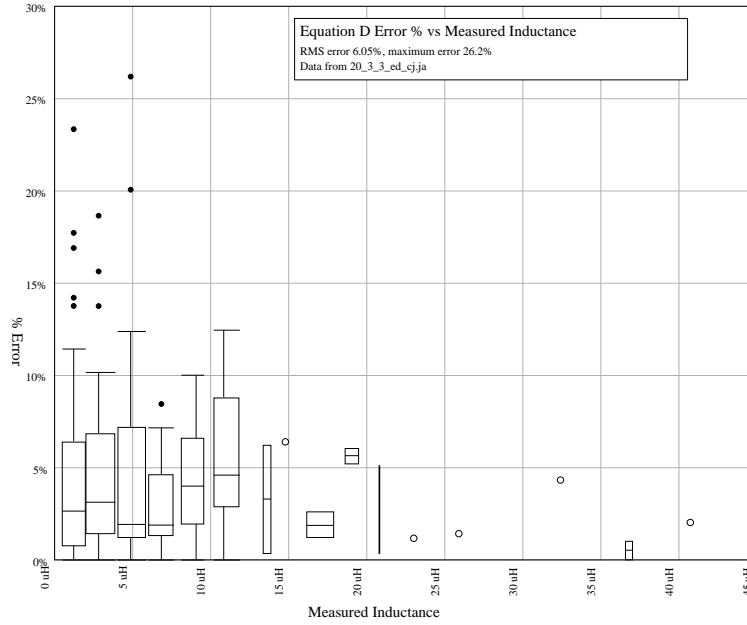


Figure 54: Equation D Tukey plot measured inductance vs error %

18.8 Equation E

Tried to use log on radius. Performs very poorly, only on gauge #22. In the code, this is called EQUATIONE.

$$\mathcal{L} = \mu_0 \frac{r_0 \log(r_1 R)(t_0 + t_1 N + t_2 N^2)}{l_0 + l_1 L + l_2 L^2 + l_3 L^3 + l_r L^4 + l_5 L^5} \quad (42)$$

18.9 Equation F

Fixed wire size for this one - #22 only. In the code this is called EQUATIONF.

$$\mathcal{L} = \mu_0 \frac{(r_0 + r_1 R + r_2 R^2 + r_3 R^3)(t_0 + t_1 N + t_2 N^2)}{l_0 + l_1 L + l_2 L^2 + l_3 L^3} \quad (43)$$

all22.csv			
Results	Mean	Worst	Seed
20_3_2_ef_cc.txt	3.38%	14.7%	321321321
nls.csv			
20_3_2_ef_cd.txt	1.69%	7.3%	32132345243

Table 15: Equation F Results

18.10 Equation G

Another fixed wire size but with length and turns. Called EQUATIONG in the code.

$$\mathcal{L} = \mu_0 \frac{(r_0 + r_1R + r_2R^2 + r_3R^3)(t_0 + t_1N + t_2N^2)}{c_d + rd_1R + rd_2R^2 + l_1L + l_2L^2 + l_3L^3} \quad (44)$$

18.11 Equation H

Another #22 gauge only. Called EQUATIONH in the code.

$$\mathcal{L} = \mu_0 \frac{l_0 + l_1L + l_2L^2 + l_3L^3}{r_0 + r_1R + r_2R^2 + r_3R^3} \quad (45)$$

19 A Different Approach

If you're winding your own coils on a different form, perhaps with different core material a simple approach works as follows.

1. Wind a few coils varying only one parameter: turns, length, radius, or wire gauge. A minimum of 3 is required, more if possible. The coils should explore both the minimum and maximum inductances you need.
2. Measure their inductance multiple times and average.
3. Measure the parameter you varied. Counting turns is easy, measuring the radius less so.
4. Perform a univariate polynomial regression on the data with a maximum power much less than the number of coils.

5. For an inductance in the range tested, vary the parameter until you get what you want. The *turntest.csv* data set example varies only the number of turns. A linear equation gives an RMS error of 4.68% for the 8 coils while a 3rd degree polynomial reduces the error to 1.52%, greatly superior to any of the equations tested.

20 A PLA Model

We concentrate on optimizing only 4 variables for our restricted equations.

Length The coil length is restricted 3 inches.

Diameter The coil diameter is restricted from 0.25 to 2 inches.

Turns The number of turns is restricted to single layer and the number that will fit on any given length.

Wire Gauge #14 to #28 is what I had.

Consider a “worst case” equation. In 46, we use a 5th order equation for both the radius and length as a real solution is likely to have an exponential component.

$$L = c \frac{(r_0 + r_1R + r_2R^2 + r_3R^3 + r_4R^4 + r_5R^5)(w_0 + w_1W + w_wW^2)(t_0 + t_1T + t_2T^2)}{(l_0 + l_1l + l_2l^2 + l_3l^3 + l_4l^4 + l_5l^5)} \quad (46)$$

L Inductance in Henrys.

R Radius in meters.

c A constant - a fudge factor.

r_i Polynomial coefficients for the radius.

T The number of turns.

t_i Coefficients for the T polynomial.

l The coil length in meters.

l_i Coefficients for the length polynomial.

W Wire radius in meters.

w_i Wire radius polynomial coefficients.

The approach:

1. For all coils collect: length, radius, turns, and inductance (average over all measurements).
2. Optimize the values of c , r_i , t_i , w_i and l_i to get the best match of L for all measured inductors (PLA only).
3. Publish the coefficients as the best solution.

Sounds easy, but suppose we try 100 values of each coefficient measured against 50 coils. That's $50 \cdot 100^{13} > 2^{64}$ calculations and it's likely that some of the coefficients might need more than 100 values tested. I can't do a brute force solution so I rely on a Genetic Algorithm for find some solution.

20.1 Build Measured Coil Database

Read all the CSV files with plain PLA, compute the average of the 30 measurements for each, convert imperial to metric units and record which file it came from. The *indbuild.java* script generates this CSV file that's read during the optimization startup.

Field	Units	Description
Henries	Henries	Measured value from coil in henries.
Radius	meters	The coil radius.
WireGauge	meters	The wire radius.
Turns	integer	The number of turns.
Length	meters	The length from first to last winding.
FileName	string	The CSV file containing this coils measurements.

Table 16: Coil measurement CSV file fields.

I printed a number of additional forms to expand on the length-radius-turns variables. These were all wound with #22 gauge wire, plain PLA, 0.125" walls with 50% fill. We build equations for 3 variables - a 3 dimensional space where the coordinates length, diameter and turns form a space who's value is inductance. Our task is to find a function of length, wire gauge, coil radius, and number of turns that accurately predicts

$$\mathcal{L} = F(\text{Length}, \text{Gauge}, \text{Radius}, \text{Turns}) \quad (47)$$

20.2 The *indefu* program

Significant double-precision floating point calculations are required for optimization. The program computes a set of coefficients from a specified range and resolution for each. These are passed to a subprocess that tweaks some of them with a smaller range and then returns the best result for the n coils we selected. The best result is the mean of the solution L_{ci} error percents.

$$g = \frac{\sum_i^n \frac{|L_i - L_{ci}|}{L_i}}{n} \quad (48)$$

Special consideration is made for values that cannot be computed (zero denominator) or zero solution.

A configuration file directs its operation. The variable names are listed with their minimum and maximum values, the resolution, the minor resolution and the number of minor resolutions to consider.

There are also a few scalar variables that can be set.

Variable	Type	Description
save	integer	The number of best values to breed against. Either 4 or 5.
maxgenerations	integer	How many generations to run.
minormax	integer	The maximum number of minor values to consider.
resultf	string	Where to put the results. If not present, results written to stdout.
seed	integer	The random seed.
noise	integer	How noisy to make the output, defaults to 1.
randomminir	double	What percentage of a random result to fiddle [0 .. 1.0].

Table 17: Scalar variables.

The general flow:

1. Select ranges and resolution.
2. Perform a run of 2000 generations. Do so on as many machines as possible with different random seeds.
3. Note any coefficients near the range limits and adjust accordingly.

4. Note any inductors that seem to be causing problems and see if they can be removed from the inductor list.
5. Repeat.

20.3 The Equation

We first removed the wire size as we weren't able to get good convergence with it. So all coils are plain PLA with #22 wire.

$$L = c \frac{(r_0 + r_1R + r_2R^2 + r_3R^3 + r_4R^4 + r_5R^5)(t_0 + t_1T + t_2T^2)}{l_0 + l_1l + l_2l^2 + l_3l^3 + l_4l^4 + l_5l^5} \quad (49)$$

It is expected that c is actually μ_0 and I removed it from the values that can be changed. Multiple runs were made usually to 10,000 or 100,000 generations. If any of the parameters in the best solution were too near the limits, these were adjusted and the runs continued. At times 4 large multi-core, multi-processor Xeon systems were attempting solutions.

Our measure of success is the mean of the errors for a particular parameter set. Each error is the difference between the measured inductance and the inductance computed by the polynomials. An alternative would be to reduce the maximum error.

The best result to date got a mean error for 84 coils of 1.77%.

coeff	1.77%	1.76%
r_0	-0.0075	-0.01005
r_1	3.28693	3.9998
r_2	290.553	295.675
r_3	291.889	557.152
r_4	-999.283	5.65763
r_5	-999.132	-1232.27
t_0	-1.3853	-1.49047
t_1	1.7135	-1.62252
t_2	0.6829	0.650825
l_0	27.8967	28.5994
l_1	743.746	782.606
l_2	2888.88	2994.88
l_3	-2720.89	-1942.07
l_4	-3682.45	-8830.3
l_5	-5989.3	-9752.52

Table 18: 1.77%, 1.76% solutions for #22 wire.

For the 1.77% solution, the worst offender was off by 6.6%. For the slightly better solution, the worse offender was off by 7.1%. As you can see from the variety of coefficients, there are many solutions.

Now that I have an approximate solution, I added a quadratic for wire size.

$$L = c \frac{(r_0 + r_1R + r_2R^2 + r_3R^3)(t_0 + t_1T + t_2T^2)(w_0 + w_1W + w_2W^2)}{l_0 + l_1l + l_2l^2 + l_3l^3} \quad (50)$$

Repeating the operation gets results in the 2.9% range, not nearly as good but probably still acceptable. The maximum error is 23.7%

coeff	2.9%
r_0	0.02805
r_1	-9.90285
r_2	-645.876
r_3	-1.28767
w_0	-0.117525
w_1	1.89535
w_2	0.98515
t_0	-1.3
t_1	1.4
t_2	0.9
l_0	8.04482
l_1	359.208
l_2	-136.438
l_3	-1845.14

Table 19: Cubics with wire size.

The static spacing experiment reverts to:

$$L = \frac{(r_0 + r_1R + r_2R^2 + r_3R^3)(t_0 + t_1T + t_2T^2)}{c + r_{d1}R + r_{d2}R^2 + l_1l + l_2l^2 + l_3l^3} \quad (51)$$

20.4 Conclusions

For a restricted range of length, diameters, numbers of turns and wire gauge, equation provides a reasonable solution.

Diameters .375" to 2" with increasing errors on the outer edges somewhat matching the error margins published for the East Tester LCR.

Lengths .25 to 3" with increasing errors on the ends.

Turns Ok except where the turns are widely spaced.

Wire Size Mostly OK except where larger gauges significantly decrease the spacing between turns. Wire gauges #14 to #28 tested.

Humidity PLA is not a good plastic for devices in widely varying humidity.

Exceed these ranges at your own risk. Polynomial curve fits rarely good predictors much beyond the range of data points.

21 The *coil* program

The coil program takes a specification and generates an OpenSCAD model for 3D printing. These were used to generate all the coils used in this experiment. The command line program takes the parameters in Table 20. The first command line argument must be the output file name.

Option	Default	Description
-radius <i>float</i>		coil radius in inches
-length <i>float</i>		coil length in inches
-turns <i>integer</i>		Number of turns
Optional		
-cylindersPerTurn <i>integer</i>	18	Number of indent cylinders for each turn.
-mindia <i>float</i>	0.012	Minimum wire diameter per turn.
-wireSize <i>float</i>	0.06	Wire diameter in inches.
-wallThickness <i>float</i>	0.125	Wall thickness of coil in inches.
-wireIndentFaces <i>integer</i>	9	Number of faces for each cylinder for indent.
-formFaces <i>integer</i>	90	Number of faces for coil wall.
-gauge <i>integer</i>		Overrides -wireSize.

Table 20: **coil** program command line options.

22 Data Files and Programs

Data is stored in comma separated variable form (CSV). Comments may appear in the files when prefixed with // starting in column 1. Typical files vary one item followed by 30 measurements of inductance or Q.

File	*	Contents
<i>1in.csv</i>	*	A series of four 1 inch coils with 5 to 25 turns, plain PLA, 0.125 walls, #22 wire.
<i>15p15.csv</i>	*	1.5" diameter, 1.5" length, vary turns.
<i>15p175.csv</i>	*	1.5" diameter, 1.75" long, varying turns.
<i>1p25in.csv</i>	*	5 coils, 1.25" length with 5 to 25 turns, plain PLA, 0.125 walls, #22 wire.
<i>1p75in.csv</i>	*	7 coils, 1.75" length with 5, 10, 15, 20, 25, 30, and 40 turns. Plain PLA, 0.125 walls, #22 wire.
<i>2in.csv</i>	*	2" diameter, 1" length vary turns.
<i>2p15.csv</i>	*	2" diameter, 1.5" long, vary turns.
<i>3inws.csv</i>	*	Inductance in μH , 3 inch coil, 10 turns, plain PLA, 0.125 walls, vary wire gauge.
<i>3inwsQ.csv</i>		Q measurement for some of the above.
<i>cplafill.csv</i>		Inductance, conductive PLA, 10 turns, #22, 1" diameter, 0.35" hole, vary fill percentage.
<i>fedia.csv</i>		Inductance, iron fill PLA, 0.125" wall thickness, 50% fill, vary diameter. See Figure 22 on page 31.
<i>fepla175.csv</i>		Inductance, iron fill PLA, 1" diameter, 0.5" hole, vary fill percentage. See Figure 9 on page 22 and Figure 10 on page 22.
<i>highaspect.csv</i> *		Inductance, high aspect ratio, diameter 1.0", 0.125 walls, spacing 0.1", #22, plain PLA, 50% fill.
<i>humid.csv</i>		Measurement 8 coils with the same parameters in humid conditions. See Figure 45 on page 51.
<i>keysight.csv</i>		See Figure 34 on page 42.
<i>turns.csv</i>	*	Inductance, low aspect ratio, diameter 1.0", 0.125 walls, spacing 0.1", #22, plain PLA, 50% fill. See Figure 7 on page 20.
<i>lspacing.csv</i>	*	Inductance, 10 turns on 1.25" to 3" plain PLA, 0.125" walls, #22 wire.
<i>p375.csv</i>	*	0.375" diameter, 1" length vary turns.
<i>p5in.csv</i>	*	5 coils, 0.5" diameter, 1" length, 5, 10, 15, 20, and 25 turns. 0.125" wall thickness, plain PLA, #22 wire.
<i>spacing.csv</i>	*	Inductance, 10 turns on 0.3" to 1.0" long plain PLA, 0.125" walls, #22 wire.
<i>pladia.csv</i>	*	Inductance, 10 turns, 1" long, plain PLA, 0.125" walls, #22 wire, .5" to 1.75" diameter.
<i>plafill.csv</i>		Inductance, 10 turns, 1" diameter, 0.35" hole, #22 wire, vary fill percentage 10% to 100%. See also <i>fepla175.csv</i> .
<i>same.csv</i>		Inductance of 8 identical coils, dry condition. See Figure 45 on page 51.
<i>samedia.csv</i>		Diameter of 8 coils in both dry and humid conditions. See Table 5 on page 54.
<i>wiresize.csv</i>	*	Inductance, 10 turns, 1" diameter, 0.35" hole, plain PLA, #14 to #28 wire.

Table 21: Captured CSV data files (* used by *indequ*).

The graphs were generated by the *graph* program [11].

File	Description
<i>diameters.graph</i>	Compare inductance of various diameters and core materials. See Figure 20 on page 29.
<i>dias.graph</i>	Error results for diameters. See Figure 21 on page 21.
<i>fedias.graph</i>	Error results for diameters with iron composite PLA. See Figure 22 on page 31.
<i>femur.graph</i>	Error results for diameters vs μ_r . See Figure 10 on page 22.
<i>manuerr.graph</i>	Generate comparison graphs between East Tester and Keysight tester. Generated by <i>manu.ja</i> .

Table 22: Control scripts generating graphs.

JA programs were used to fit polynomials to some data.

File	Description
<i>3inws.ja</i>	Accuracy of wire gauge approximation.
<i>diameters.ja</i>	Build graphics for diameters and errors. Uses <i>fedia.csv</i> and <i>pladia.csv</i> .
<i>feur.ja</i>	Calculate μ_r from equation 4 on page 13 for the iron composite PLA.
<i>indbuild.ja</i>	Build the CSV file for <i>indequ</i> program to help optimize equation 46 on page 68.
<i>manu.ja</i>	Compare Keysight values against East Tester values. See Figures 33 to 36 starting on page 42.
<i>plafill.ja</i>	Calculate μ_r from equation 4 on page 13 for plain and conductive PLA.
<i>spacing.ja</i>	Fit curve to the coil spacing data <i>spacing.csv</i> and <i>lspacing.csv</i>

Table 23: Polynomial curve fitting and error computation.

The following are some of the STL and gcode files used to generate the forms.

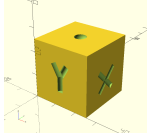
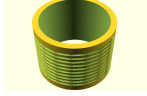
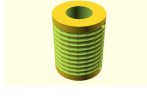
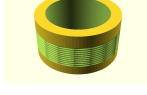
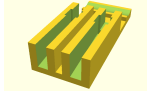
Picture	File	Description
	<i>cube.*</i>	Permittivity test panel. Modified before printing for fill density.
	<i>dnpee.*</i>	Various diameters from 0.5" to 1.75", 10 turns, #22 wire.
	<i>exx.*</i>	1", 10 turns, various wire gauges.
	<i>tight1.*</i> <i>tight8.scad</i>	- Diameter 1", 10 turns, varying coil length from 0.3 to 0.9, #22 wire.
	<i>stand.scad</i> <i>stand.stl</i> <i>stand.gcode</i>	East Tester Kelvin probe test stand.

Table 24: Printable designs.

23 Some General Comments after 12 Months

- Don't use transparent plastic for coil forms, you can't see the wire indents. Black is not a good choice either.
- No matter what color you use, you'll be sick of it by the time you finish.
- Label each coil. After a while, they all look the same. A series of dots and dashes on the edges (Morse code) is easy for small coils.
- You're working near the resolution limit on low cost 3D printing. On my printer you can't expect a 0.03" hole to be at least that size. The trough for smaller wires will be problematic.
- Winding countless coils is somewhat like knitting. If you drop a turn, you have to back up or start over.

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